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ABSTRACT

This is a first-grade manual for teachers using SMSG elementary school text materials. For each chapter background information is given; for each section, objectives, vocabulary, materials, suggested procedures, and further activities are listed. Chapter topics include: (1) pre-number experiences; (2) recognizing geometric shapes; (3) building number concept; (4) pre-addition and subtraction experiences; (5) numbers and the number line; (6) comparing geometric shapes; (7) addition and subtraction; (8) place value and numeration; (9) introducing linear measurement; (10) extending addition and subtraction; and (11) previews and extensions. (MP)

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MATHEMATICS FOR THE ELEMENTARY SCHOOL

Book 1

Teachers' Commentary

(preliminary edition)

SE 021 962

MATHEMATICS FOR THE ELEMENTARY SCHOOL

Book 1

Teachers' Commentary

(preliminary edition)

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Chapter I

PRE-NUMBER EXPERIENCES

Background

Why sets? We know that most of an elementary mathematics program is concerned with arithmetic, and that this begins with counting and addition and subtraction. Why does this book start out with these ideas? Why start with the idea of sets? These are questions that teachers may ask before beginning to teach this unit, and such questions demand an answer.

One answer to these questions is that a set is what you count. We may count 6 cats, or 6 boys, or 6 ice cream cones. Yet the concept of the number 6 does not depend on cats, or boys, or ice cream cones. The thing that is common to all of the collections of objects which we count is that they are sets. The notion of set will recur throughout the entire mathematical program and it is wise to introduce the term early and to use it effectively in building the notion of number. We shall also see that the simplest descriptions of addition and subtraction--indeed the classical descriptions--are in terms of manipulation of sets of physical objects.

The basic notion is that of a set of physical objects. Thus we may consider the set of all children in a room, a set (or a bunch) of grapes, or a set of books. A set is completely specified when its members are specified, and the members of a set belong to it. Thus John may belong to the set of pupils in the classroom, and the teacher is a member of the set of people in the room (at least, most of the time). There need be no relation between the members of a set. We may even occasionally run into a set consisting of a few oranges, an apple, and some umbrellas.

We say that one set is a subset of another if each member of the first is a member of the second. Thus the set of girls in a classroom is a subset of the set of children, and the set of all tricycles is a subset of the set of all toys. The set consisting of John and Mary is not a subset of the set of all girls because John does not belong to the set of girls. We shall need the notion of subset when we consider the manipulation of sets which leads to subtraction.

There is one very special set. Suppose a club is going to have a program and members are volunteering for the various committees. Then a number of sets are automatically created: the set of volunteers for the refreshments committee, the set of volunteers for the decorations committee, and so on. But what of the set of volunteers for the clean-up committee? We all know that often this is the empty set. The empty set has many other descriptions. It is the set of purple cows, or the set of green-haired boys. There is just one empty set. Later we will say that 0 is the number of members of the empty set.

Another curious sort of set is a set which has just one member. We need to consider sets of this kind because we are going to use the number one. If Alexander, Balthazar and Constantine are the members of a set and Constantine and Alexander leave in a huff, the set which remains has just one member. We will eventually relate this situation to the number sentence: 3 - 2 = 1.

Let us review the notions we have just introduced by considering a little problem. Let us attempt to list all possible subsets of the set consisting of an apple and an orange. It is pretty clear that there are two subsets which have just one member each: the set consisting of the apple and the set consisting of the orange. Are there other subsets? What of the empty set? Is it true that every member of the empty set is a member of the set consisting of an apple and an orange? If not, then some member of the empty set must fail to belong to the apple-and-orange set, and this is impossible since the empty set has no member. We must therefore agree that the empty set is a subset of the set consisting of an apple and an orange, and in fact that the empty set is a subset of every set. (We will later relate this to number sentences like: 5 - 0 = 5.) Finally, we may ask whether this apple-and-orange set is a subset of itself. Is it true that every member of the set consisting of an apple and an orange belongs to the set consisting of an apple and an orange? But this is obviously true, so we agree that the apple-and-orange set is a subset of itself. In fact, the same consideration shows that every set is a subset of itself. (We will relate this fact to number sentences like: 7 - 7 = 0.) Mathematicians have expressed their general feeling that it's sort of improper for a set to be a subset of itself by agreeing that a *proper* subset of a set is to be a subset which is different from the whole set. A proper subset of a set always has fewer members than the set.

The next task, after we have established the notion of set, subset, empty set and set with one member, is to prepare for the concept of number. To understand how we do this let us consider a related problem. How do we convey the notion of red? Suppose a child is beginning to learn about color, and we want to explain the color red. We would surely point to, or touch, all of the red objects around, trying to lead him to conceive of the color red as the property which is shared by all of these objects.

What is our conception of the number 7 ? It is surely the property which is shared by all of the sets having 7 members.* But this raises a difficulty. Can we talk about all of the sets having a certain number of members without having the notion of number? Even simpler: can we decide when two sets have the same number of members without knowing how to count them? Can you discover whether there are the same number of boys and girls in a room without counting? Can you find out whether there are the same number of cups and saucers on a shelf without counting? The solution to our problem is clear. We can pair each boy with a girl, and we can pair each cup with a saucer. In fact, *there is a way of deciding whether two sets have the same number of members without knowing anything about number*. This process is our next concern.

Suppose we are given two sets which we wish to compare. We may *pair* each member of the first set with a member of the second set as long as possible. If we run out of members of the first set but not of the second we know that there are *fewer members in the first set than the second*; if both sets are exhausted at the same time we say the first has as many

---

*We might go even further and *define* the number 7 as the collection of all sets having 7 members. Using the ideas which are explained in the next two paragraphs we can state this definition without using the notion of "7 members", and say: 7 is the collection of all sets equivalent to:

△ △ △ △ △ △ △

members as the second, or that the sets are equivalent*; and if the second set is used up first we say there are more members in the first set than in the second. We can describe these possibilities in a slightly different way. We try to set up a one-to-one correspondence between the sets. If we succeed, the sets are equivalent. If we find a one-to-one correspondence between the first set and a proper subset of the second (part, but not all of the second) then there are fewer members in the first set or more members in the second set.

All of the foregoing is preparation for the concept of number and the notion of inequality of numbers. There is also background which is needed for the later development of addition and subtraction. In this first unit we try to familiarize the children with the necessary manipulations with sets and to establish the vocabulary for these manipulations.

The join, or union, of two sets consists of all objects which belong to either or to both of the sets. In your classroom, the join of the set of boys and the set of girls is the set of all children. We may represent two sets, say on the flannel board, and think of joining the first set to the second, or of joining the second set to the first. Of course the same set results (we say that joining is commutative). This fact is the foundation for our later discovery that addition is commutative.

Suppose we are given a set and a subset and we remove the subset. The set which is left is the remainder set; it consists of those objects which belong to the original set but not to the subset. If we remove the set of boys from the set of children in the classroom the remainder set is the set of girls. We later relate this to the fact that the number of children minus the number of boys is the number of girls.

Finally, the result of removing a subset and then joining it to the remainder set is always the original set. It is also true that if we join one set to another and then remove it we get the original set. We notice these facts about sets now because they will later show us that addition and subtraction are, in a certain sense, inverse operations.

---

*Equivalent is not synonymous with equal. Two sets are equal if and only if they have the same members; i.e., the sets A and B are equal if and only if every member of A is a member of B and every member of B is a member of A. For example: the set consisting of the president, the vice-president, and the secretary of state is equivalent to the set consisting of the British Prime Minister, the Chancellor of the Exchecquer, and Minister of War, but these sets are certainly not equal.

Teaching Notes

These first lessons are conducted without using numbers. The length of time devoted to sets before the introduction of numbers will depend on the maturity and previous experience of the class. Those children who have had kindergarten experience with sets will probably need only a brief review; on the other hand, children not having had kindergarten might spend more than one class period on a single idea like that of joining sets. It is hoped that set vocabulary will be used outside of the arithmetic period and that the teacher will find opportunities to reinforce the ideas of the mathematics program in other classroom activities. Already many children are familiar with such collections as a set of dishes, set of silverware, bag of fruit, sack of potatoes, collections of sea shells, rocks, butterflies, stamps, etc., group of children, family, box of toys, etc.

Many kinds of material can be used to illustrate the use of the word set: blocks, toys, books, and other tools of the classroom; flannel board materials; objects from the child's home environment (sets of dishes, chairs, etc.); and the children themselves. A sequence for teaching early set vocabulary might be:

1. Developing the meaning of the word "set" through experiences in describing sets.
2. Developing understanding of the word "member" by listing orally the members of sets and determining whether things are or are not members of given sets.
3. Developing the meaning of the word "subset" by experiences showing that subsets may be formed in many different ways.

4. Showing that a set may have only one member or may, indeed, have no member. The concept of a set with one member may seem easy but the teacher should make sure that children understand it. There is usually no difficulty in teaching the notion of the empty set; children find this very intriguing.
5. Giving actual experience in joining one set to another and making it possible for the child to discover that the order of joining two sets does not affect the union of the sets.
6. Giving children opportunities to remove a subset from a given set and to become familiar with the term "remainder set" for the set that is left.

A teacher should not feel confined to the suggested activities of the following lessons. However, suggested lesson plans are given in order to indicate ways of developing the objectives of the lesson. Teachers may wish to follow part of the plans, or all of the plans, or they may combine activities from two or more lessons into a single teaching period. Activities of the same or of different sorts may be used later to reinforce the ideas.

I-1. Set and member

Objective: To introduce the ideas of set and member and the use of these terms to describe things we observe around us.

Vocabulary: Set, collection, member

Materials:

1. A variety of set materials, such as: books, blocks, pencils, crayons, sheets of paper, paint brushes, scissors, game boxes, puzzles, beads, pegs, balls, etc.
2. A variety of flannel board objects: animals, fruit, stars, trees, story-book characters, stick figures, etc (Exclude geometric shapes at this time.)
3. Magazine pictures of collections: family, automobiles, telephones, clothing, food, toys, planes, trains and any other appropriate illustration.

Suggested procedure:

Children are familiar with sets of dishes, silverware, blocks, tinker toys, etc. Use these examples when first discussing the term set. After the general introduction of various sets, focus attention upon further describing the collections. (We are aware that there's a major difference between Susie's playhouse dishes and Mother's fine chinaware--price-wise if nothing else! Yet both are referred to as a "set of dishes.") The following may be helpful as a guide for the introduction to sets.

> How many children have helped to get the table ready for dinner?
>
> What do you put on the table? (plates, glasses, silverware, etc.)
>
> We usually have to be careful with the set of dishes because dishes can break. What do you have in your set of dishes at home, Angie? (plates, cups, saucers, etc.)
>
> Angie's set of dishes is a collection of several objects--plates, cups, etc.

While Angie lists the objects, the teacher may have flannel board materials ready and place them on the flannel board as each is mentioned. This should keep the class attention and provide reinforcement.

Discuss the collection and ask for other collections. Some class replies may include collections of stamps, coins, sea shells, butterflies, etc. Refer to each of these as a set as well as a collection.

Ask the children to look around the room and find some sets. At first the children will probably hesitate because of the new vocabulary, etc. Help them by describing sets and having them guess what they are. Note the game "I'm Thinking of a Set" under "Further Activities" of this lesson. Some of the following sets may be described: chairs, tables, books, paint brushes, crayons, science-table displays, game boxes, etc. These set descriptions should be more explicit than the earlier ones.

> We have many books in our room. How can we talk about these books? (Very good: a set or collection of books in our room)
>
> Let's look at this set of books in the room. What books do you see? (set of reading books, library books, etc.)
>
> Are the books on my desk part of the set of books in our room? (Yes) Tell me as many things about this set of books as you can: tall books, red books, reading books, etc.

Throughout these exercises emphasize thorough set description by suggesting ideas, possibly in the form of question and class reply. For example, if you wish to have Billy bring you the set of red pencils on your desk, the directions must be explicit. He may bring any set of pencils in the room if the set isn't clearly defined. It must be clear that the set desired is the set of red pencils on the teacher's desk.

After the children have practiced describing sets, the teacher may introduce the idea of member of a set. The word "member" should be explained and understood by the children before it is used in conjunction with the expression "member of a set." The children are members of the class; they belong to it. Some children may belong to church and recreation groups. Soldiers are members of the Army, etc. Each child is a member of his family. The family is a collection of people; each person in the family is a member of it. The following lesson is taught in conjunction with "the family" presented in the children's reader. Use illustrations and make references to this family to clarify the discussion.

> We read a story about a family today. The family is a collection of people. Is that a set? (Yes)
>
> Who were the members of this family? (use flannel board material or pictures to suggest mother, father, etc.)
>
> Each one in the family is an important member of it. Just as each person in the family is a member of that set, each thing in any set called a member of that set (e.g., children are members of our class.)
>
> Let's think of some of the sets we found in our room and name their members. (Review the sets previously discussed and decide what their members are. First describe the set. Then name the numbers of the set.)

After discussing the members of a set in general, point out that some things are not members of some sets. For instance, the books are not members of the set of paint brushes, Johnny is not a member of the set of girls, etc.

The ideas of set and member may be used throughout the day in classroom happenings. This vocabulary can be used as incidental opportunities arise.

Further activities:

1. Art activity: Have children draw pictures of sets and orally name their members. Suggestions may include: child's own family, storybook family, toys, things that grow in the yard, etc.

2. Oral reading of a story dealing with a family or set of some sort.

3. Initiate a class project of creating a science-table display with collections of rocks, sea shells, leaves, butterflies, etc.

4. Game: The teacher says, "I'm thinking of a set that looks like - - - (describe set)." The pupil says, "Is your set the set of - - -?" If the answer is correct, the child becomes the leader and thinks of a set. If he is incorrect, further description is given until the set is identified.

5. The daily calendar work can be used in conjunction with the vocabulary introduced. (e.g., the set of days in the week, the set of days we go to school, the set of weeks in the month, etc.)

NOTES

I-2. Subsets of sets

Objective: To introduce the idea of subset of a set

Note: The notion of a set with one member does not occur until Lesson 4 of this chapter. However, children may discover this set naturally in discussing subsets (for example, the set of all adults in the classroom). The teacher should take advantage of such an occurrence to discuss sets of one member further.

Vocabulary: Subset; review set, collection, member

Materials:

1. Variety of materials and flannel board objects such as listed in Chapter I, Section 1
2. Colored construction paper; several sheets each a different color
3. Jar of multi-colored beads
4. Rhythm instruments
5. Set of play dishes

Suggested procedure:

The concept of a subset of a set may be developed as follows:

> Will each member of the set of children in our room raise one hand?
>
> Do all the boys have one hand up? (Yes)
>
> Do all the girls have one hand up? (Yes)
>
> Everyone put his hand down.
>
> Now, let's have the girls stand. Is Jane a member of the set of girls? (Yes) Repeat this question about several girls; then ask the question inserting a boy's name.

Is Michael a member of the set of girls? (No) Michael is not a member of the set of girls. Of what set is he a member? (The boys.) Will the boys stand? (Repeat as with the girls.)

We have seen that there is a set of children in our room. We also know that there is a set of boys and a set of girls. We say that the set of girls is a subset of the set of children because every member of the set of girls is a member of the set of children. The set of boys is a subset of the set of children because every member of the set of boys is a member of the set of children.

(Ask the boys to stand again. Then ask three to raise their right hands.) Tom, Billy, and Johnny have each raised their right hands. Are these three boys members of our set of boys? (Yes) However, they have their hands raised. We call the set of boys in our class with their hands raised a subset of the set of the boys in our class.

Repeat the above process with the girls: set of girls and set of girls holding books; or, the set of girls and the set of girls wearing white shoes.

This exercise can be developed further using the entire set of children and finding subsets.

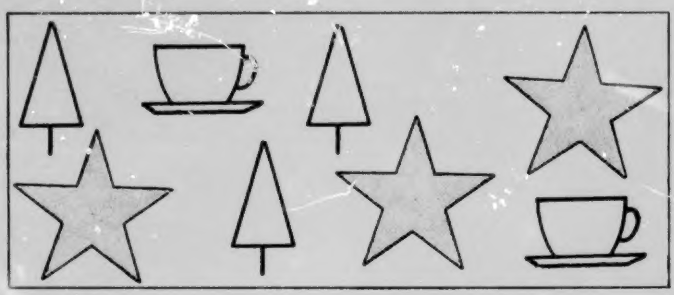
At this point the teacher can use a chalkboard or flannel board to develop further the idea of subset of a set.

Place a set of fruit on the flannel board. Talk about the set of apples as being a subset of the set of fruit, the set of pears as a subset, etc.

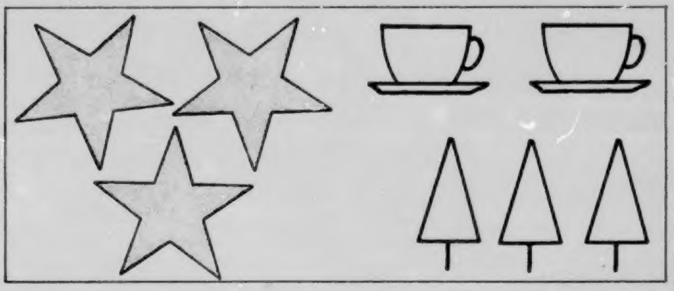
Caution: It is necessary to identify the set first before talking about a subset of that given set. We cannot start with a subset and then think of a set.

Further activities:

1. Place a set of objects on the flannel board (stars, trees, etc.) Have the children describe this set.



Ask a child to come to the flannel board and place all like members together.



Do we still have the set we started with? (Yes) What subsets have we found in the original set? (cups, stars, and trees.)

2. Place several blocks of various colors (or crayons, sheets of construction paper, etc.) on a table. Have the set described. Invite the children to identify the various subsets (red blocks, yellow blocks, etc.) Ask if the red blocks are members of the set of blocks; if the yellow blocks are members of the set of blocks, etc.

3. Talk about the set of dishes used at home. Describe the original set and identify various subsets (e.g., cups, saucers, plates, etc.). A demonstration with play dishes should help to add interest and understanding.

4. Other activities for developing the idea of subset are:

   a. Identify various subsets within a set of multi-colored beads.

   b. Identify the subsets within a set of rhythm instruments. (e.g., drum, triangles, etc.)

   c. If classroom "helpers" are used, all of the helpers can be considered as a set. Identify the subsets of this set. (e.g., the helper(s) who waters the plants in the room.)

   d. Refer to the buildings in a town as a set. Identify the subsets. (e.g., houses, offices, hotels, stores, libraries, etc.)

   e. In addition to separating the class into boys and girls, you may separate into subsets of: children with curly hair; boys with red on; girls with green; everyone with tie shoes; those who want to work in certain work areas during free time, etc.

Again, the teacher may find many opportunities during the day to reinforce the concept of a subset of a set.

NOTES

I-3. The empty set

Objective: To develop the idea of set with no members

Vocabulary: Empty set

Materials:
1. Small objects such as beads, paper clips, keys, or like material to be placed in children's pockets
2. An apron, dress, jacket, or piece of clothing which contains at least one pocket (Teacher will wear this.)

Suggested procedure:

The concept of the set with no members can be developed very well through a discovery game. On the day of the lesson the teacher should wear a piece of clothing having at least one pocket. Some time before the activity is to begin, select at least three children who have pockets and place a set of small objects in one pocket of each. Include objects such as suggested above or others that are of interest to your class. When the experience is to begin, have one of the children empty the contents of his pocket on the table. The class can then describe the members of the "discovered set." The following may be helpful:

> Let's look at the things in __________'s pocket. What can you tell me about the members in this set? (paper clips, beads, etc.)
>
> Are there any more members in your pocket, __________? Your pocket is now empty.

Repeat the above for each child previously chosen. Make it amusing and fun for the children by using the game idea. After each child has emptied his/her pocket, select one child to come up and empty the teacher's pocket which has nothing in it.

> What are the members of the set of things in my pocket? (No members)

> You mean my pocket is empty. There are no members in it. When there are no members in a set it is called the empty set.

Review the sets previously placed on the table and ask if any of these sets were empty. Since they each had members, there was no empty set. The teacher can continue the game and idea as she sees fit.

The following activity may be used as a variation of the one for introducing the empty set. Whereas the first procedure actually involved the removal of the sets, the following is a "Game of Pretend." Prepare four boxes before the class period. Select three children and tell them that they are to pretend that these things came from their pockets last night. The fourth box is empty; representing the teacher's pocket last night.

> We are going to pretend that each of these boxes contains the things that Jamey, Michael, Julie, and I took out of from our pockets last night. Jamey, show the class what you had in your pocket. (e.g., doll dress, paper clip, bobby pin, and eraser.) What are the members of Jamey's set? (Class describes.) (Progress to the other children's sets in like manner.)
>
> Now let's see what I had in my pocket last night. Johnny, will you come up and name the members in my set, (Child reaches around and finds box empty.) What did you find, Johnny? (Nothing, no members, etc.) You mean my box is empty. I guess I had nothing in my pocket last night. It was empty. When there are no members in a set, it is called the empty set.

Further activities:

1. Manipulative game: Select several objects that the children are able to identify by touching (Indian corn, book, pine cone, pencil, paper clip, paint brush, rock, leaf, etc.) For example, place on a table a box of pine cones, a box of pencils, a box of rocks, and an empty box. Blindfold a child and have him come to the table where these four boxes have been arranged.

   > I'll tell you what you may find: a set of pine cones, a set of pencils, a set of rocks, and a set of books. But one of these sets is empty. Can you tell me after you feel each set which one has no members? (the set of books)

2. In some games, such as Duck, Duck, Goose, it is possible to point out the empty set.

3. Make reference to such amusing sets as the set of elephants in the room, the set of lions at the store, etc. The pupils' imaginations will extend the activity as far as the teacher desires.

NOTES

I-4. Set with one member

Objective: To introduce the idea of set with one member

Vocabulary: No new terms

Materials: Physical sets, chalkboard illustrations, flannel materials

Suggested procedure:

By this time the class and teacher have worked together in forming and reorganizing the children into subsets of various types: boys, girls, boys with blue shirts, girls with pig tails, etc. After these concepts have become part of the class thinking and vocabulary, the concept of one member set can be easily introduced if it has not been considered in previous discussions. The following may be helpful.

> Today, let's have all the girls wearing red stand. (Continue with a variety of ideas to instill interest.) (The teacher should now be seated at her desk.) Now, will the set of boys stand? set of girls stand? set of teachers stand? (Reactions at this point will appear from the class.) Let's all be seated.
>
> Yes, I am the only member of the set of teachers in this room (assuming that there is not more than one teacher). Your sets have many members but my set has just one member. Can you think of any other sets in our room with just one member? (The set of pianos, the set of teacher's desks, etc.) Sets can have many members such as the set of the children in our school; or they can have just a few members or just one member or no members at all. Is the set of elephants in our room a set with one member? (No! There aren't any elephants in here. It's the empty set!)

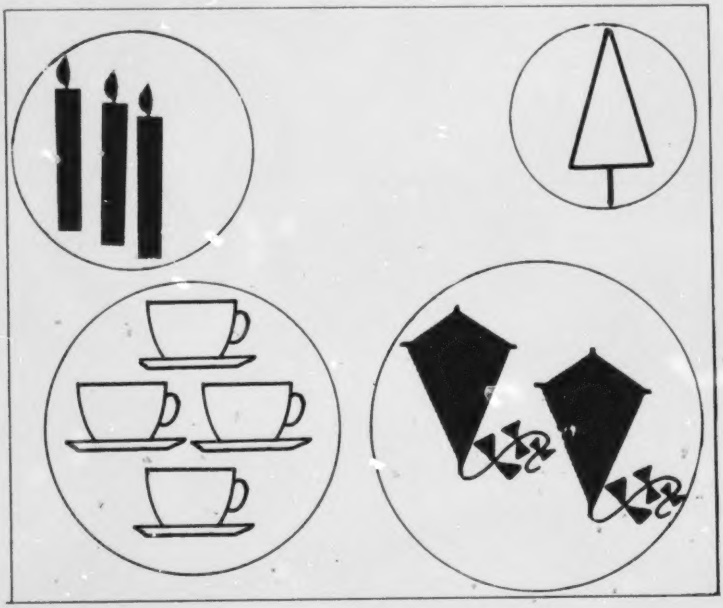
The following activities may be used for further development of the objective.

Further activities:

1. Flannel board objects placed on flannel board in random order. Have children separate the collection or set into subsets. One subset should have just one member. The children can identify this set as a set with one member.

2. Talk about the family again and separate it into the subsets of mother, father, and children; mother and father each being one member sets. You may extend this to include something which would be the empty set.

3. Chalkboard illustrations such as those below can help to stress concept.



4. Begin with talking about all the children in the school as one set. Break this set down into subsets of each class or classroom. These sets, in turn, have various one-member subsets: the set of all girls named Sharon Logan, the set of all boys named John Kelley, etc.

I-5. Joining and union

Objective: To introduce the operation of joining and its commutative property.

Vocabulary: Join, union.

Materials: Flannel board materials, beans, milk bottle tops, buttons, spools, etc.

Suggested procedure:

After the children are familiar with the ideas of set, member, subset, empty set, and set with one member, they should be ready to understand the concept of join as it is used in connection with sets. Some will have understood the word as they have heard it used in other situations.

The teacher might start with a demonstration at her desk or at a low table where all can see. On one side of the table, place a set of ten or fifteen various sized buttons and on the other side a set of flannel board materials (trees). Each set should contain too many members to be counted quickly as no counting is wanted at this time.

After the sets have been identified by the children, proceed somewhat as follows:

> I am going to move this set of buttons over to join it to the set of trees.
>
> When we join sets, we say we have a union of the sets.
>
> What are the members of this union? (the buttons and the trees that were members of the two sets)

Move the set of buttons back to its original position.

> Now I will move the set of trees over to join it to the set of buttons.

> What do we call the result when we join two sets? (The union of the sets)
>
> What are the members of this union? (The buttons and the trees)

The teacher should follow the same procedure using the flannel board and materials. In both, stress the fact that one set is joined to the other to form a new set. Again stress join and call the result the union of the sets.

Further activities:

Continue with other examples of joining sets using suitable materials. In each case emphasize that

(1) a new set is formed when one set joins another,

(2) each member of the new set is a member of one of the sets joined,

(3) the order of joining does not change the new set.

> We have our set of books about animals here at the library table. On my desk is the set of new animal books that I got at the library today. John, will you bring the set of new books and join it to the set here at the table?
>
> Would the union of these sets be different if we took the set of animal books from the library table back to join the set of books on my desk? (No.)

Using your set materials already listed, see that each child has sets on his desk, one set on each side. Again have too many members in each set to be counted quickly.

> Put your right hand on the set of beans on your desk. Now move this set over to join the set of spools.
>
> What do we call the result when we join two sets? (The union)

Have the first set moved back to its original position and move the set on the left over to join the set on the right. Again speak of the union of the sets.

Other opportunities for using the word "join" will arise in the course of the daily activities.

> Bill's father is coming this afternoon to help Bill show you the fossils they found this summer. Tom suggested that we ask the other first grade to join us at that time.

NOTES

I-6. Removing and remainder set

Objective: To introduce the ideas of removing and remainder set and the use of these ideas

Vocabulary: Remove, remainder set

Materials: Flannel board materials, e.g., apples, bananas, oranges, cherries; individual set materials

Suggested procedure:

On the flannel board place a set of fruit containing six or seven members of at least three kinds of fruits. Ask the children to describe the set. It may be "a set of fruit" or "a set of apples, bananas, and cherries."

> Let's pretend we're going on a picnic today. We will each take a sandwich for ourselves and something to share with the whole group. On the flannel board we have the set of fruit for the picnic. Dick, you will take some fruit to share. Will you remove the subset of apples? What set do we have remaining here on the flannel board? We call this set the remainder set. The remainder set is the set that is left after we have removed a subset from the original set. The members of the remainder set must always have been members of the original set.

Ask the child to return his subset to the original set. Another child should then be asked to choose a subset of fruit to take to the picnic. This may be the same subset or a different one. Again stress that the remainder set is the set that is left after we removed a subset from the original set.

Further activities:

To reinforce the concepts of remove and remainder set, this same type of lesson can next be done with each child using sets from his individual set materials at his desk.

After the children have this concept well in mind more than one subset may be removed from the original set.

Rhythm instruments of several kinds are put together in a set. The set is described and then one by one subsets are removed from the set and the remainder set identified each time.

NOTES

Chapter II

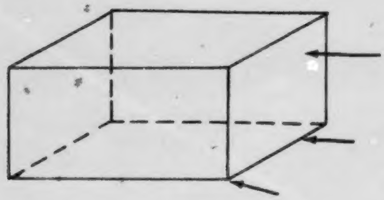
RECOGNIZING GEOMETRIC SHAPES

Background

The subject matter of geometry is really a set of idealizations of objects familiar to us in our every day experience. Thus we shall ultimately speak of a point as an exact location, an idea suggested to us constantly by such different things as the tip of a pencil, the corner of a block, or a small dot on a sheet of paper. Similarly a line segment is a path between two points which is suggested by a tightly stretched string, a crack in the floor, the edge of a block, and the like. A line is a segment extended indefinitely in both directions. A plane is the ideal flat surface suggested by such things as the face of a block, the top of a desk, or a wall. Similarly a triangle is a closed path which is a union of three line segments with only the endpoints in common. A rectangle similarly is a closed path in a plane which consists of four line segments which has all four angles right angles (which in turn must be defined).

The purpose of this chapter however is not to make these abstractions but to develop the child's perception of space. He is asked to observe, touch, examine, and make observations about actual physical objects of his experience. He learns to discriminate shapes and to name some of the simple ones, specifically circle, rectangle, and triangle.

The approach is through three-dimensional objects such as balls, cans, boxes, blocks, and the like. These are examined and classified according to such obvious criteria as whether the object has corners and edges (like a block) or edges but no corners, (like a can) or neither (like a ball). In examining a block or box he is led to notice the flat faces, the edges where two faces meet, the corners etc., as noted in the next diagram.

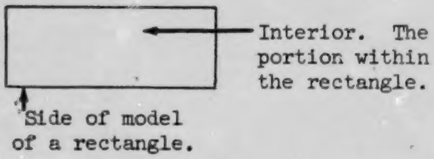


There may be interest in noticing how many faces meet at a corner and how many edges meet at a corner. It is not particularly intended to emphasize the technical names of the figures, but such names as sphere, cylinder, and (for the above) rectangular prism may be used if the children are interested.

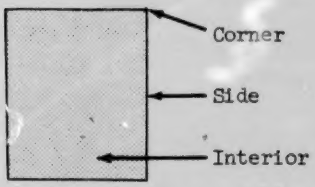
After the introduction with three-dimensional objects, attention is directed to certain shapes which lie in a plane, namely the circle, rectangle, and triangle. A word of explanation on the use of these terms is in order. In colloquial speech these words are often used in two senses. Thus one may describe a child's hoop as being a circle and may also call a clock face a circle, though it is perfectly clear the two ideas are different. In the case of the hoop we are looking only at a curve. In the case of the clock face we clearly are considering a curve _and_ _all_ _the_ _points_ _inside_ _it_. A similar situation arises with the other figures where we often use the word rectangle to describe both a lawn and the fence around it. For clarity we wish to distinguish these two senses. Specifically the words "circle," "rectangle," and "triangle" will be used to mean the curve. (It may be noted that the word "curve" is often used, as here, to describe any kind of a path so that triangle and rectangle as well as circle are special kinds of curves.) Thus the child's hoop is a good model of a circle, a narrow picture frame with the picture removed suggests a rectangle, and the usual musical instrument called a triangle is a good model of what we wish to suggest by the word. The corresponding figures consisting of these curves and their interiors are called regions.

Thus the clock face is called a circular region since the edge is a circle and an ordinary sheet of paper is a rectangular region since its edge is a rectangle.

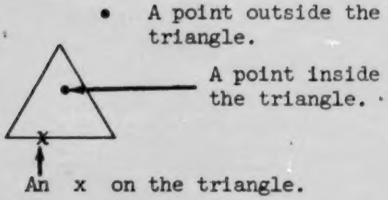
In order to make the desired association of the terms it is suggested in Section 2 that the physical models be of wire or other suitable substance which definitely show the curve but not its interior. Naturally when the children are asked to look around and find examples of figures which have the "shape of a rectangle," etc. many of them will show the region, but by calling attention to the edge, running the finger around it, etc. the attention can be directed to the desired idea, without making a major production of it. The idea that a rectangle is only the curve is further reinforced by discussing the placing of dots inside or outside or on a rectangle drawn or laid on a flat surface. The diagrams below indicate the essential ideas.

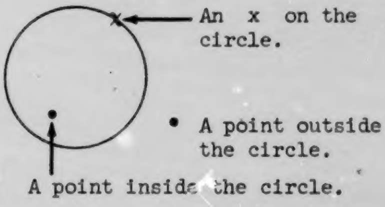

Interior. The portion within the rectangle.
Side of model of a rectangle.

The edge suggests the abstraction which is the rectangle.


Corner
Side
Interior

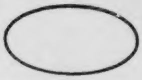
Rectangular Region, the curve and its interior. In geometry any path between two points is called a curve. A rectangle is composed of segments of four lines that intersect in a special way.


A point outside the triangle.
A point inside the triangle.
An x on the triangle.

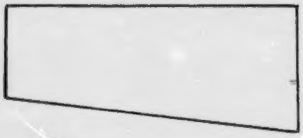

An x on the circle.
A point outside the circle.
A point inside the circle.

In Section 3 of this chapter the difference between rectangle and rectangular region, etc. is explicitly called to the child's attention. He is asked to notice how they are alike and how they are different, and the teacher uses the word "region." Actually a large number of the models that will be used will really be models of regions--for example most flannel board models will be of this type. For this reason, in order to avoid a labored vocabulary when the attention is really on something else, it is suggested that the teacher may wish to make an "agreement" with the children to use the word rectangle as a short way of talking about a rectangular region also but with the "understanding" that the rectangle is really only the edge. Thus the teacher could ask a child to put four green triangles on the flannel board. If this is done the teacher should periodically refresh the "understanding" by asking where the triangle really is.

In both Sections 2 and 3 the primary goal is to help children note certain distinguishing characteristics of circle, triangle and rectangle by working with familiar models of each. For example, they should be led to note that a triangle has three sides and three corners. A circle on the other hand has no corners at all. This distinguishes it clearly from triangle and rectangle. However the child should also be led to notice the difference between a circle and another oval such as this by observing that a circle is the same in all directions while the figure here looks bent out of shape. It is longer one way than the other.



The rectangle is more difficult. One obvious characteristic is that it has four sides but we also want to lead to a discrimination of rectangles from other quadrilaterals. The teacher should seek to lead the children to a recognition that for a rectangle the four corners all "look alike" while for other quadrilaterals such as the following they do not all "look alike."



Thus a rectangle is to be recognized as a four-sided figure with the corners alike. Quite probably there need to be several experiences of this type, perhaps first with only rectangles, triangles, and circles to distinguish from each other and later with other models such as ovals and quadrilaterals from which the desired ones are to be selected.

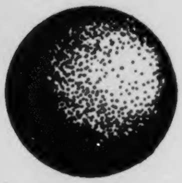
In the case of both triangles and rectangles care should be taken to show several shapes. For example not all triangles shown should be equilateral. In the case of rectangles some but not all the models should be squares. The deliberate intention here is to have the squares recognized as special rectangles. A little later we shall ask the children to identify from a set of rectangles, the subset of squares but the term is not introduced here. If, as is not unlikely, some child injects the term the teacher can agree and indicate that some rectangles are indeed squares.

II-1. Familiar three-dimensional figures

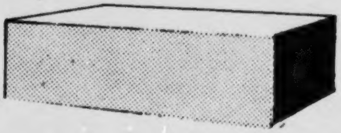
Objective: To lead children to observe distinguishing features of spheres, rectangular prisms, and cylinders

Vocabulary: Shape, round, face, edge, corner, inside, outside, surface

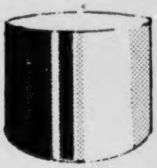
Materials: A table on which there are familiar objects (at least 15) of various shapes: balls, boxes, blocks, plastic containers, and the like. These should be restricted to objects that can serve as models of spheres, rectangular prisms, and cylinders.



Sphere



Rectangular prism



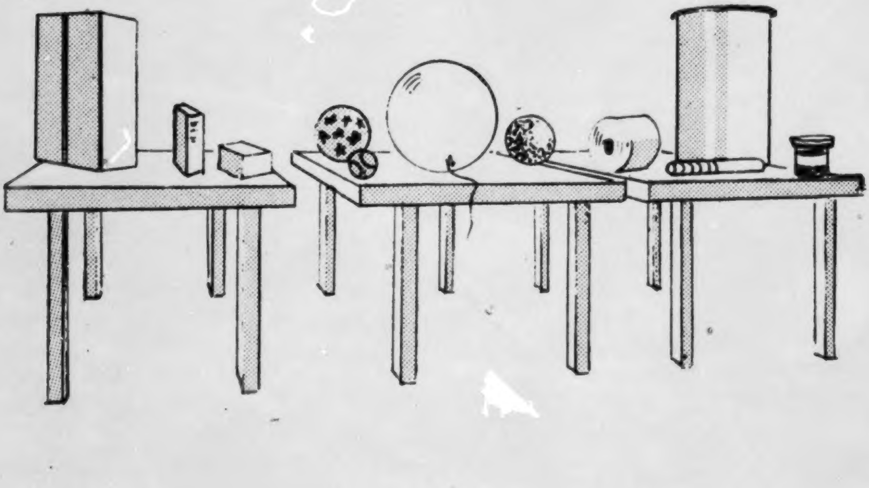
Cylinder

Suggested Procedure:

This exploratory lesson is designed to direct children's attention to certain geometric attributes that may be associated with objects shaped like spheres, rectangular prisms and cylinders. There should be a sufficient number of objects (varied in color and shape) so that all children have opportunity to handle and discuss the objects. They should run their hands over surfaces, along edges, etc.

You may begin this lesson by designating desks or sections of a table on which children are to place objects that have some kind of likeness. Ask one child to place an object in one of the places. Ask another child to select a second object. If it can be placed with the first object because of some likeness, he should place it with that object. He should give his reason for placing the objects together. If he does not think it should be placed with the first object, he may place it on another desk and explain in what way these objects are different. The classification has been established at this point; and when the other children place objects in the various sets, they should use this same classification. You may find that the first sorting is done according to color or size or use of the object or material from which it is made, etc. Let the children continue the classification by using six or more objects. As each object is placed with a set, discuss with the children whether or not it belongs with the other objects in the set.

Start with all objects in one set again and tell the children to think of another way to sort the objects. Let children develop several classifications, but if shape has not been used as a basis for sorting, then you should develop this. You could do this by first placing a ball on one desk, a box on the next desk, and a can on the third. Then you could select another object and ask the children where it should be placed. If a child gives the correct placement, ask why he put it there. The placement might be correct but the answer incorrect in which case you could say that this had not been the reason, but that the object was in the right place so it could stay there. (For example, a child might place a red bead with a red balloon.) You could then ask if any other child could think of a reason why that was the right place for the particular object. If the children did not yet supply the reason (same shape) you could proceed to another object. The activity should finally result in some such arrangement as that pictured below.



After the sorting is completed the children should identify what the objects in each set have in common. Their description of the sets may be: objects like boxes, objects like balls, objects like spools. You should help to develop the awareness of each of these sets by further describing the boxes as having edges, flat sides, (faces), and corners; cans as having edges but no corners; and balls as having neither edges nor corners.

Pupil Page 1: Use a crayon to mark the first picture and the object in the row which have the same shape.

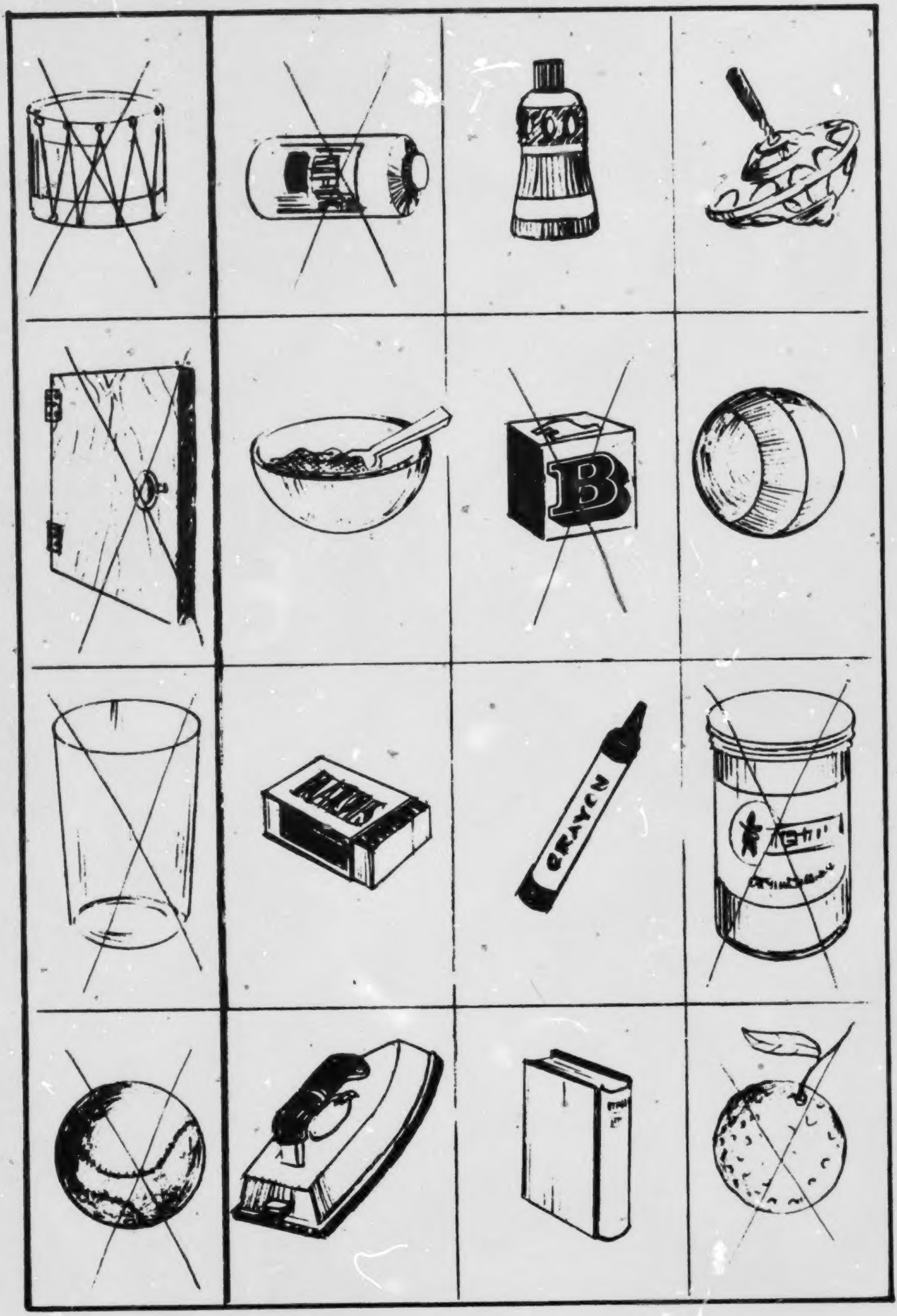
Further Activities:

1. Ask a child to put his hands behind his back. Then, place in his hands an object shaped like one of the three kinds in this lesson. Ask him to tell what set it belongs to. Continue with other children and other objects. It would be advisable to include objects (of the same shapes) which had not been used in the earlier sorting.
2. Have children identify other objects in the room that could be placed in one of the three sets. Children may wish to bring from home pictures of objects that can be placed in one of the three sets. Have them tell why each object can be put in that particular set. This not only helps to identify these shapes but it also provides opportunity for associating a picture of a three-dimensional figure with the figure itself. These pictures may be arranged on a bulletin board, or in a scrapbook, etc.
3. If children ask the names of the shapes that they handle, supply these names whenever you can. Although introduction of such names as "rectangular prism," "cylinder," and "sphere" is not the purpose of this chapter, some children are interested in new words and will take pleasure in hearing them.

4. Read to the children such books as:

   Berdney, A Kiss is Round
   Roberts, The Dot
   Schlein, Shapes
   Wolff, Let's Imagine Thinking of Things

NOTES


B
CRAYON

II-2. Circle, rectangle, and triangle

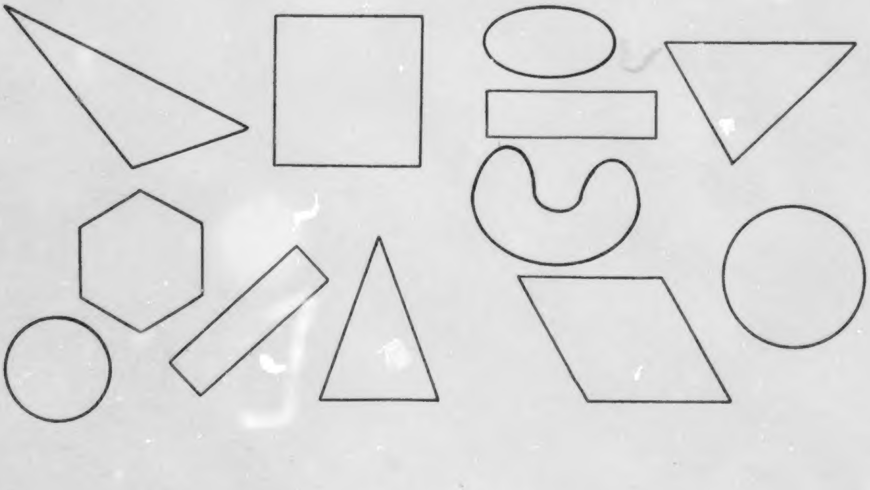
Objective: To develop concepts of cirlce, rectangle and triangle

Vocabulary: Circle, rectangle, triangle

Materials: Models of circles, triangles, and rectangles. These can be made from pipe cleaners or wire or starched string. Triangle from rhythm instruments, picture frames, and circular embroidery hoops might also be used; and for triangles and rectangles, rubber bands stretched around pegs on pegboards make good models. (Cardboard or construction paper sheets of circular, triangular, or rectangular form will be used shortly (Section 3) to represent circular, triangular, and rectangular regions, but in the present section we prefer models that suggest only the edges of such regions.) Should include two or three circles of different sizes, two or three rectangles, where one of them is a square, two or three triangles of different types; chalk and string for drawing circles on the chalkboard, straightedge; narrow strips of paper.

Suggested Procedure:

Before the lesson draw within children's reach some geometric figures on the chalkboard. The figures below are illustrative.



Display models of a triangle, circle and rectangle (such as triangle from rhythm instruments, embroidery hoop, and picture frame).

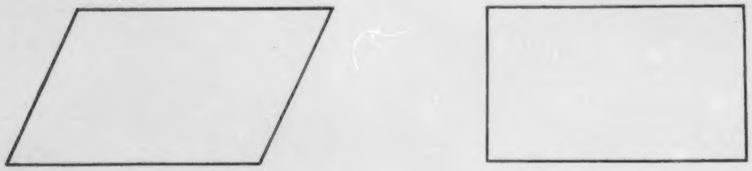
> We want to look for things shaped like these. (Indicate models). Let's first look for things like this (indicate triangle and move it from others for focus of attention). See the pictures on the chalkboard. Which ones are like this? (Indicate model) Do you know what we call these figures? (After the triangles have been marked, supply the name if not known by any of the children and that these also have the same name as the musical instrument.)

You may also want to look for objects about the room that are like the figures and model displayed. If children identify any object shaped like a triangle, ask what represents the triangle. Ability to make a distinction between triangle and triangular region will be developed in the next section.

Continue the lesson by calling attention to the model of the circle. Proceed in much the same way as was done with the triangle.

Then direct attention to the picture frame (or whatever model have selected for the rectangle). Identify the figures on the board that are like this (indicating rectangle). Some children may be reluctant to mark the square as a rectangle and will comment, "But that's a square." Explain that a square is a special kind of rectangle just as a lollipop is a special kind of candy.

After children have marked figures like the rectangle, call attention to two figures like these:



Ask children why the first figure is not called a rectangle. (Important! Please include the reasons children give in your report on this section.)

Pupil Page 2: Have the children mark the three figures at the top of the page by first coloring the triangle and then coloring all other figures on the page that represent triangles. (Remember that only the line is colored. The interior should not be colored.) It might be well to put the figure on the board and show how to color using colored chalk. For example,

Using another color, identify the circles, and then using still another color, identify the rectangles.

This next activity is designed to help get a better understanding of the characteristics of the circle, triangle, and rectangle by identifying the figure, inside the figure, and outside the figure. Again, display a model of the triangle, the circle and the rectangle on the flannel board, chalkboard or table. (Model must be in plane) Place an object inside the figure. If a small toy animal or car is used, discuss with the children whether this animal can get out without climbing over the picture frame, hoop, etc. Then talk about where objects would be placed if outside the circle, rectangle, etc. Ask children to place objects as directed:

Put the car on the hoop.
Put the car inside the frame.
Put the stick inside the triangle.
Put the stick on the frame.

Continue with this kind of activity as necessary to help children develop these ideas. Instead of objects, you may wish to use drawings and chalkmarks on the chalkboard.

Pupil Page 3: As a follow-up of this activity, use this pupil page. First ask children to indicate the figure shaped like a circle, like a rectangle, and like a triangle. Then give them instructions for putting colored marks inside, outside, and on the figures.

Place a red dot on the rectangle.
Place a red dot outside the triangle, etc.

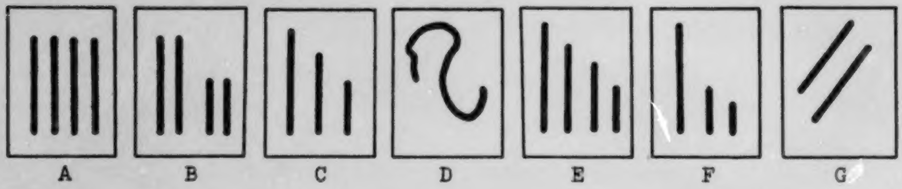
We further develop children's understandings of the characteristics of these figures by having them make models. You may wish to delay this part of the section until later in the year, or do only a part of it, or use it only with more able children. Make adaptations for your particular class.

Use sets of narrow strips of flannel or pipe cleaners on the flannel board, sticks on a table, etc., such as 3 sticks of different lengths, 3 of the same length, 2 of the same length, 3 where the union of the two shorter sticks is less or equivalent to the third, only 2 sticks, 4 sticks, etc. With each set of materials ask a child to make a triangle. Note that models may be placed in various positions, that sometimes a triangle cannot be made, and that sometimes there is not enough or too much material.

You may wish to stop with this activity. Should you wish to continue with similar experiences for the circle and the rectangle, provide a set of sticks for making a circle, finding that one cannot be made. Ask for suggestions as to the kind of material needed. If children do not suggest a piece of yarn or string, do so. Let them display the general idea of how a model of a circle could be made.

Then use 4 sticks--sets where the 4 are of the same length, 2 of one length and 2 another, 4 different lengths, only 3 sticks, only 2 sticks, 5 sticks, etc. Sometimes children may form a figure even with 4 sticks where they do not have a rectangle. Try to get them to tell how the sticks must be placed to make a rectangle. (This again is asking the children to express their ideas of the difference between a rectangle and a quadrilateral.)

Provide each child with a set of materials as suggested below. Ask him to try to make one of the figures--a circle, a triangle, or a rectangle.

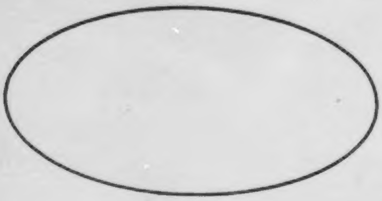


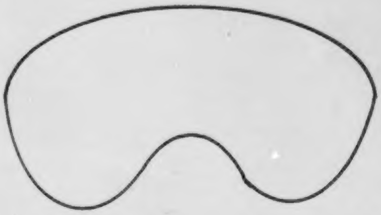
A. four sticks (or similar material) of equal length
B. four sticks--two of one length and two of another length
C. three sticks which will form a triangle
D. a piece of string
E. four sticks--all of different lengths
F. three sticks which will not make a triangle
G. two sticks

They may find that they cannot make anything, or as in the case of the string, they can make more than one item.

Pupil Page 4: As a follow-up of this activity, tell the children that each box on the page contains a picture of a set of materials like those they have been using on the flannel board and at their desks. Direct the children to look at the box in the upper left corner of the page. Place a set of materials like those pictured in the box on the flannel board, or on their desks. Ask the children to think which of the figures could be made. If a child indicates that he knows which shape can be made, ask him to rearrange the strips to make that figure. When he has finished, ask the children to put a check on the figure at the bottom of that box which shows the kind of figure made. If none of the figures can be made, instruct them to put an X inside the box. At your discretion, children may be allowed to continue the page by getting material and working alone at their desks and marking the shape that they were able to make. Some children may be able to identify the figure without the use of materials.

Now help children learn to pick out circles, triangles, and rectangles when the figures must be looked at more critically. The precise verbalizations are not important, but it is hoped that the child can be led to a working characterization of the three shapes involved. For example, he should reach the understanding that a closed figure with three straight sides is a triangle. A circle not only has no corners, but it looks the same from all directions. If someone turned it, nobody could tell the difference. Hence the first figure below is not a circle since it is clearly longer one way than the other and the second figure has a dent in it and surely cannot be called round. Some child may even make the observation that a circle has a center like the hub of a wheel. The rectangle is a little more difficult to describe but it is hoped the children can be led to see that for a rectangle the four corners look alike while for other four-sided figures the corners do not look alike. Thus a rectangle is any four-sided figure whose four corners look the same. Use a model of a circle, triangle, and rectangle. Ask the children to look at them very closely. Ask the children to shut their eyes while you turn one of the models. Ask the children to look at the models and tell which one you turned. Being assured that you turned one each time, they observe that one cannot tell if a circle has been turned. This may also be true of the rectangle or triangle, depending on the amount of rotation.





Pupil Page 5: First have the children mark the three shapes at the top of the page by outlining each shape with a different color. The children then use these as a guide to mark all other shapes of that kind in the same way. If any object is not a circle, rectangle, or triangle direct the children to place an X in it.

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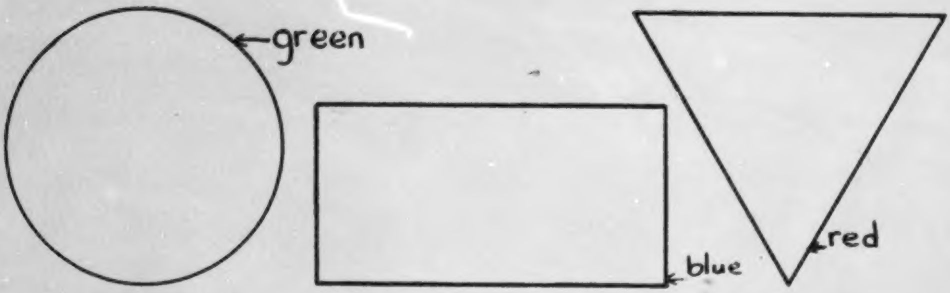
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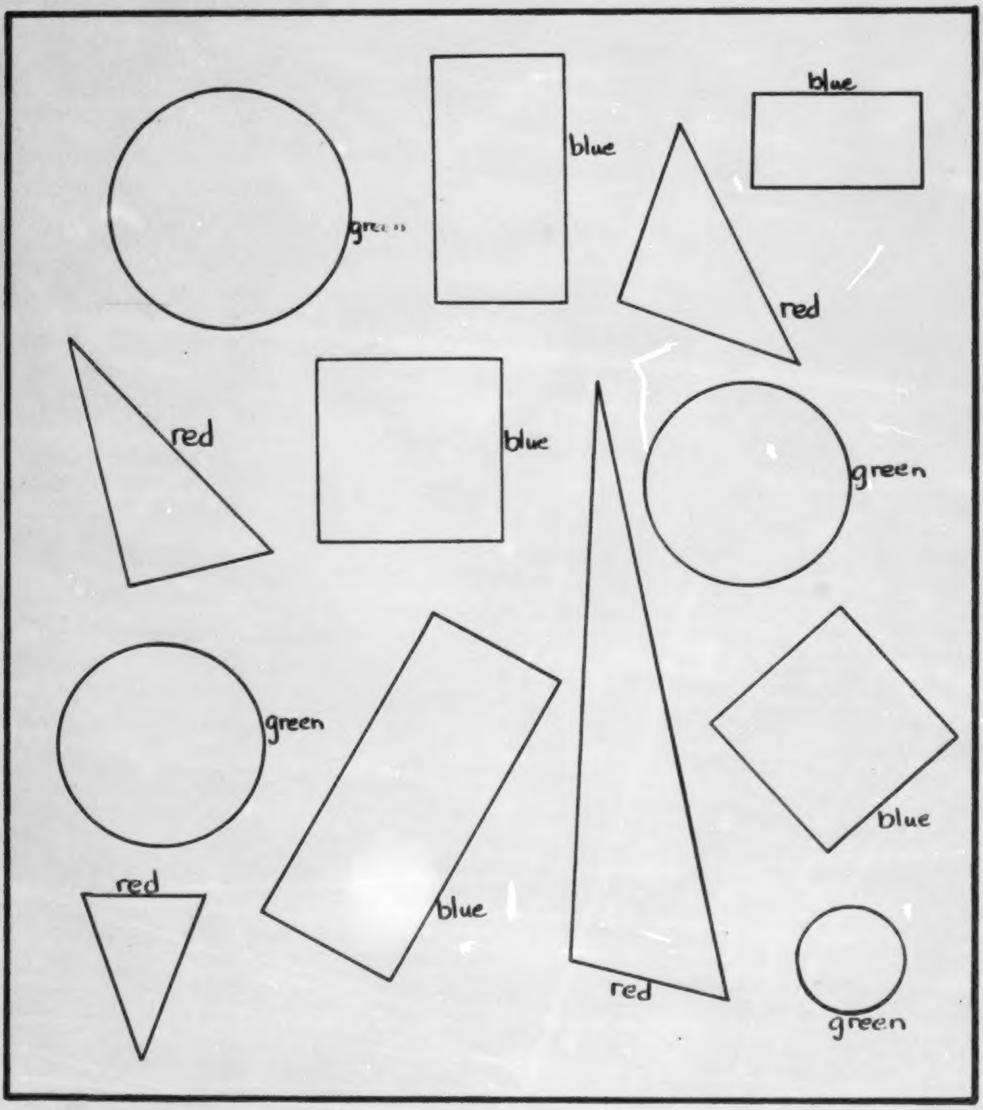
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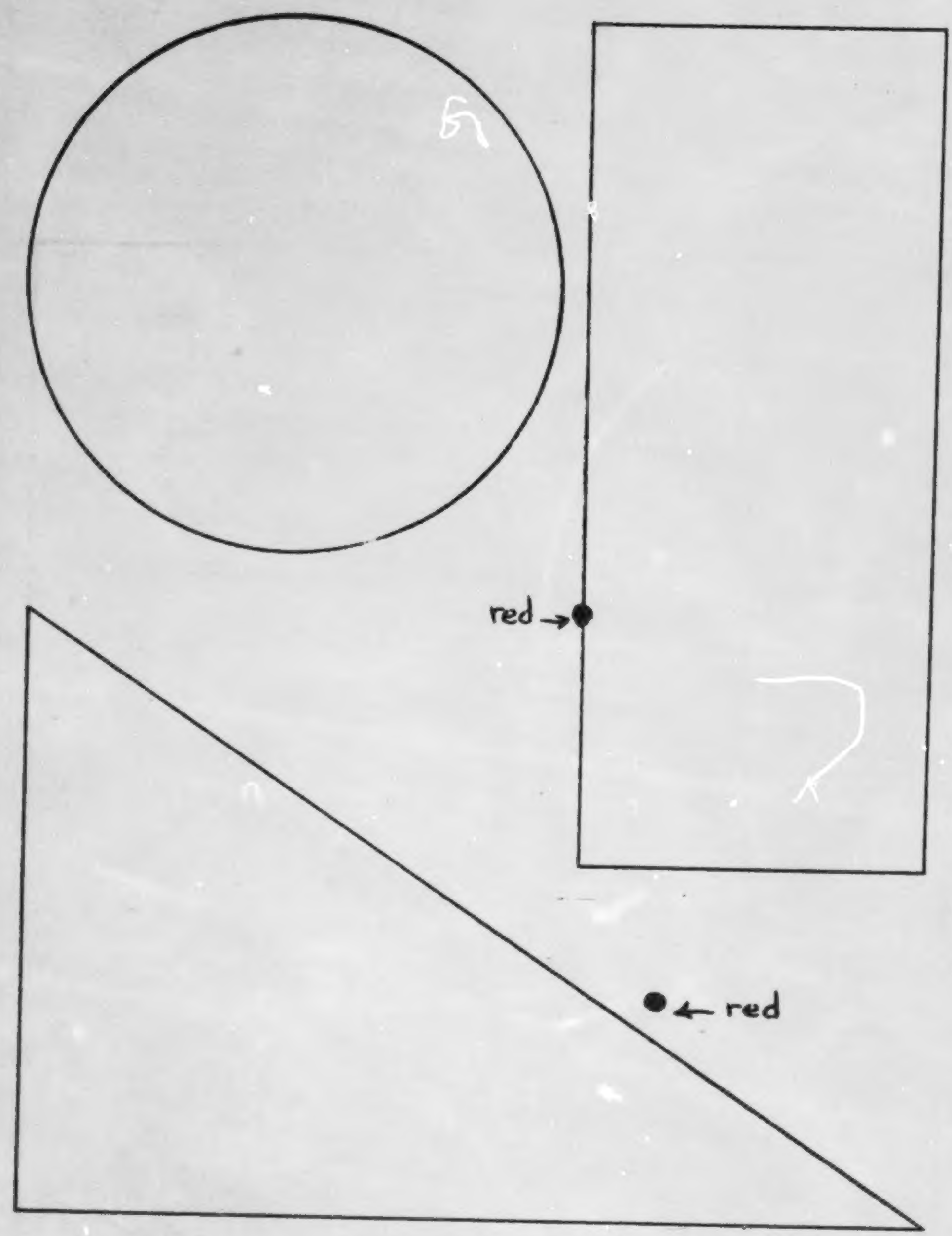
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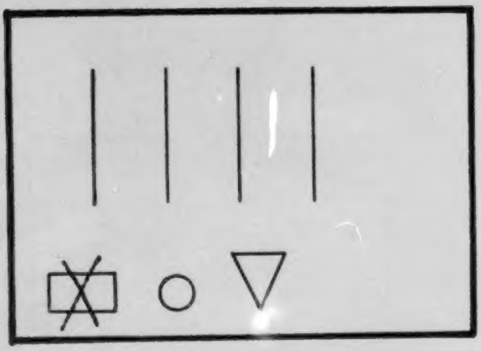

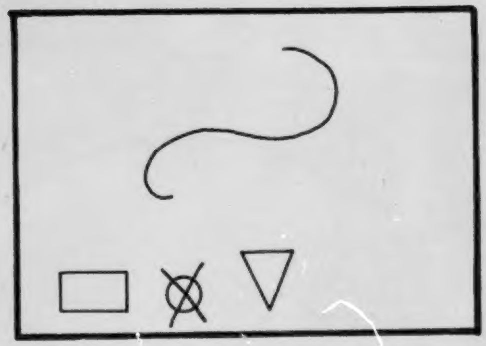

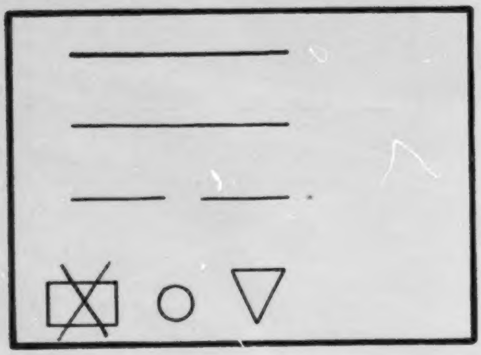

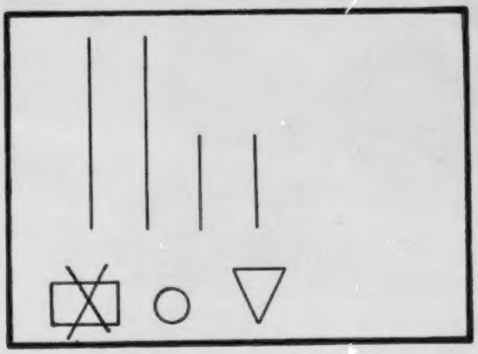

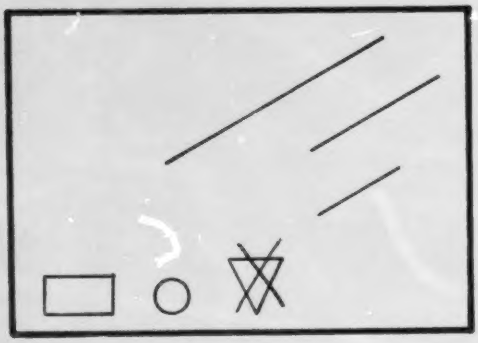

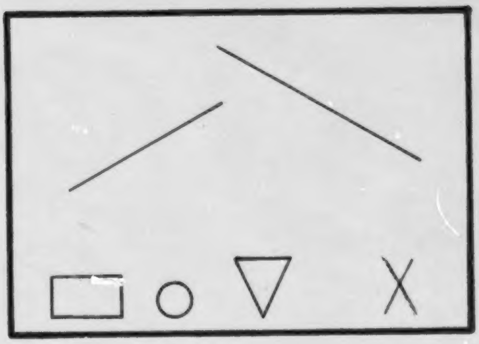

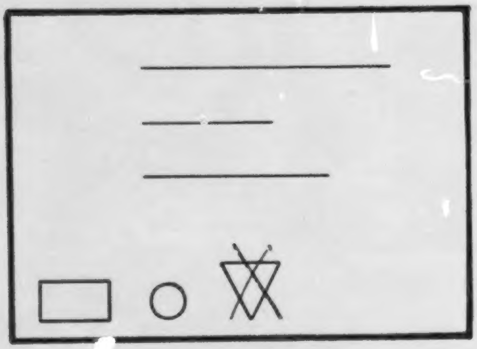

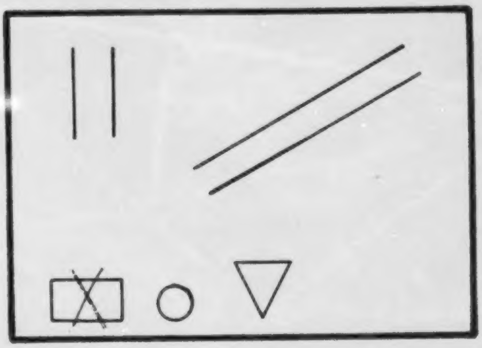

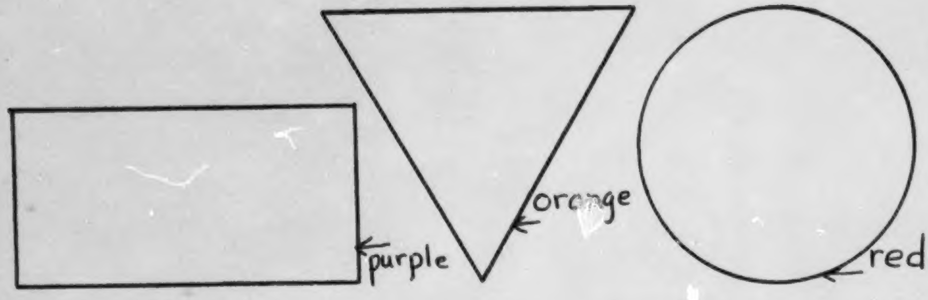
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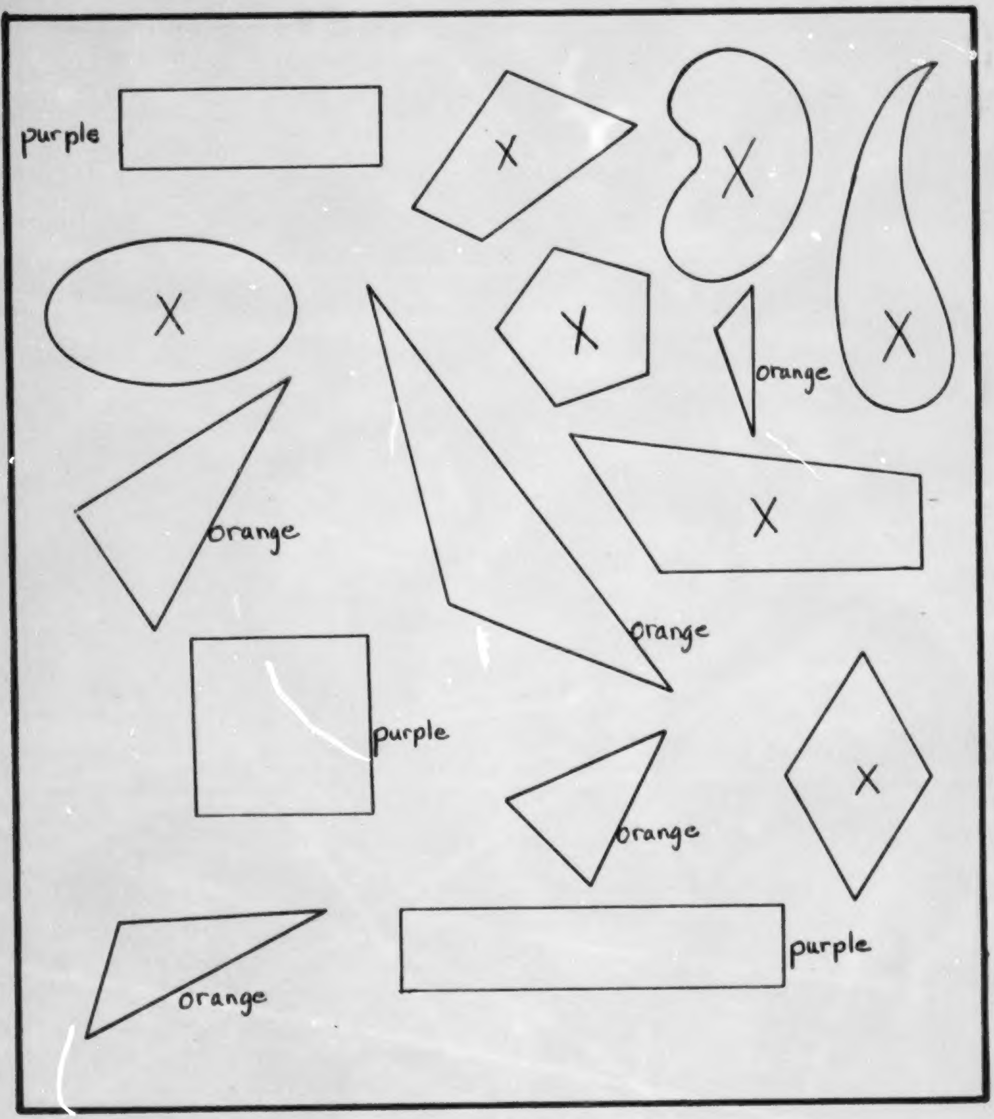
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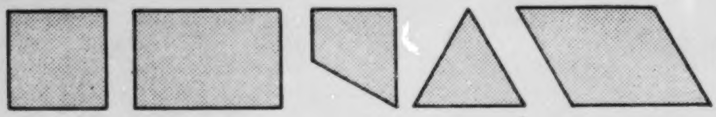
II-3. Circular, rectangular, and triangular regions

Objective: To extend concepts of circle, triangle and rectangle

Vocabulary: Circular region, rectangular region, triangular region

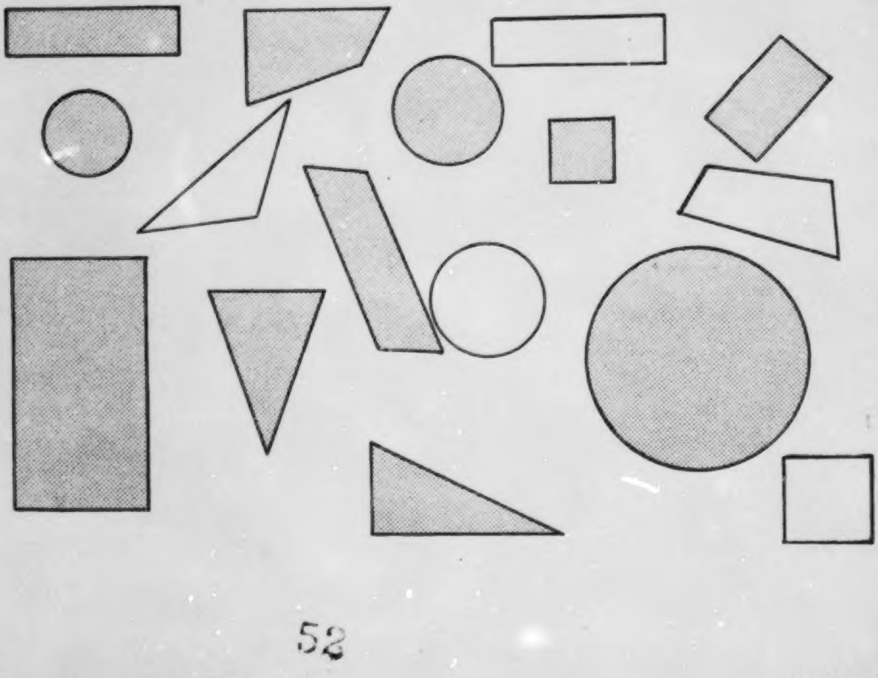
Materials: Materials used for Section 2

Models of regions bounded by circles, squares, triangles, of various types, rectangles including some squares and one or two quadrilaterals that are not rectangular. These models may be made from felt or construction paper. Running a marking pen of contrasting color lightly along the edges calls attention to the edges. The outline should be wide enough to be seen but yet not too wide.



Suggested Procedure:

Before the lesson place on the chalkboard some paper models of geometric figures. Such figures are illustrated below.



The essential purpose of these lessons will be to help children make a distinction between circle and circular region, triangle and triangular region, rectangle and rectangular region. Although the distinction may not be seen clearly by some of the children, it seems worthwhile to introduce the possibility of it into their thinking at this time. The distinction will become clearer in recurring experiences throughout the program.

Show the children models of the figures used in Section 2. Have children name these models. Then show the children a model of a circle and a circular region. Ask how they are alike and how they are different. (Alike in shape. The edge of the felt or paper figures is like the wire or string model and so on. The inside of the felt figure is "full.") Tell the children that objects like the felt cutout have a long name. They are called circular regions. The edge is a circle.

Conduct a similar discussion of rectangle, rectangular region, and triangle, triangular region. Call attention to the drawings on the chalkboard. Ask the children which are shaped like rectangles. Which is a picture of a rectangle? a rectangular region? and so on.

Place an assortment of cutouts on the flannel board. Include all the shapes mentioned in the materials. Ask, "Who will show us a rectangular region? You may take it from the set of cutouts on the flannel board." Continue until all the circular, rectangular, and triangular shapes have been removed. For the figures left, mention that some day we will learn about them. If the children ask what they are called, tell them that these figures are quadrilaterals (should they be quadrilaterals), etc.

Pupil Page 6: Instruct the children as follows:

1. Let's look at the set of pictures in the top row. What pictures are circles? Draw a ring around each circle.
2. (Continue similarly. Row 2, triangle. Row 3, rectangle. Row 4, rectangles.

3. Look at the bottom row. Find the circle. Make a dot inside the circle.
4. Now find the triangle in the bottom row. Make a cross on the triangular region. (Show X on the chalkboard.)
5. Now find the rectangle in the bottom row. Make a check mark outside the rectangle. (Show ✓ on chalkboard.)

Further Activities:

1. Place parquetry blocks in a bag for a game of identifying shapes. If blocks are not available, shapes cut from tagboard or cardboard may be used. Children take turns. Each reaches into the bag without looking and identifies the shape of a block by feeling it. He describes it, for example, as "The block is shaped like a triangle." Then he brings out the block. If the other players agree that he is correct, he places the block in front of him. Otherwise he returns it to the bag. At the end of the game, the child having the most blocks is the winner. It is necessary, of course, to establish the rule that each child must have the same number of turns. Children can make tally marks to keep track of their turns.

2. Start *Our Big Book of Shapes* with a page for each of the shapes--rectangular region, triangular region, and circular region. Paste a model cut from construction paper at the top of each page. Children may cut pictures from magazines and paste them on appropriate pages. Do not hastily reject a child's selection as incorrect; inquire. Some aspect or detail that escapes your attention may have been seen by the child.

3. Give children models of geometric regions cut from colored construction paper. They may assemble the shapes into "pictures" of animals, people, boats, buildings, tree forms, and so on.

4. Provide parquetry blocks and design blocks for children to use in making designs, pictures, etc. Further intuitive understanding among geometric figures can be developed by such experiences. The intent of this chapter is to introduce concepts and vocabulary rather than

to have children "master" the content. However, the simple evaluation sheet entitled Shapes will give information of help to you as you pla[illegible] informal experiences to reinforce and clarify ideas.

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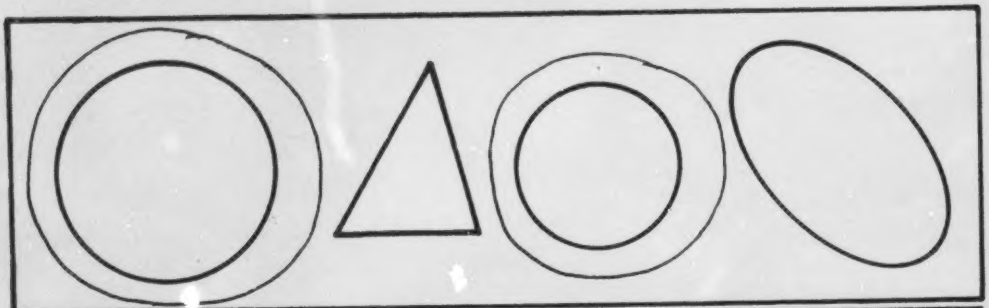
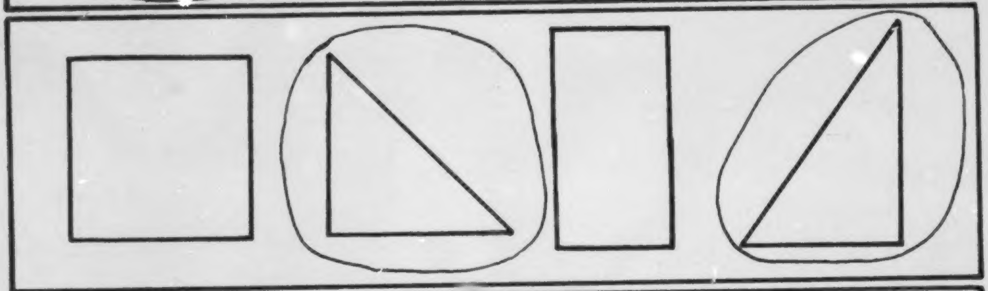
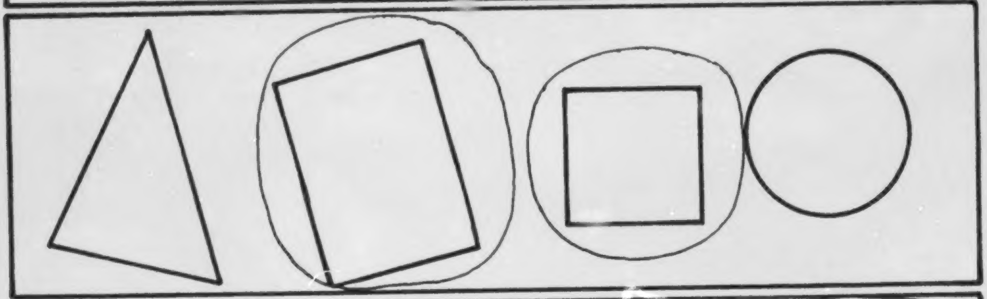
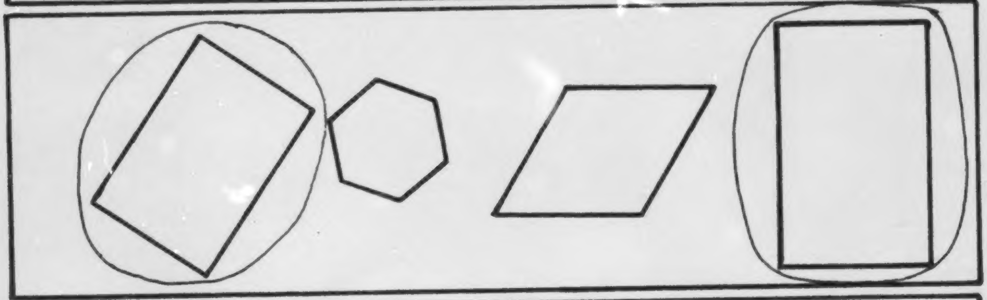
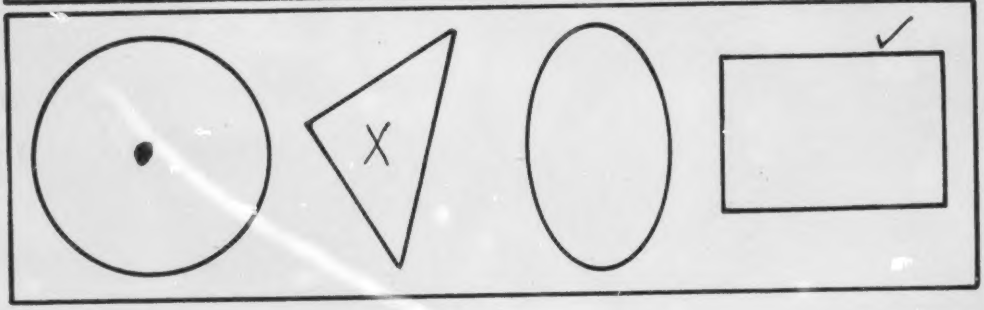










6

Chapter III

BUILDING NUMBER CONCEPT

Background

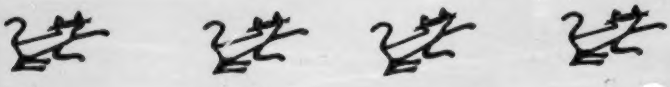
You may want to review the background of Chapter I since many of the notions developed here were introduced informally there.

The fundamental idea underlying the concept of number is that of comparison of sets. We can discover without counting, whether one set contains more, fewer, or as many members as another set. We do this as follows: we form pairs, using one member from the first set and one from the second set, until the members of at least one of the sets are all gone. When this pairing off has proceeded as far as possible we observe one of three occurrences: we have run out of members of the first set but not of the second, in which case we say the first set has fewer members than the second; we have run out of members of both sets simultaneously, in which case we say that there are as many members in the first set as the second, or that the sets are equivalent; or we may have run out of members of the second set but not of the first, and we say that the first set has more members than the second.

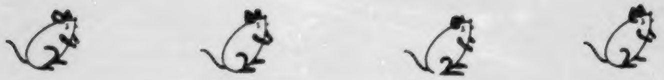
For example, if we want to compare the set of boys in the classroom with the set of girls, we can form couples as long as possible. If we run out of boys first, there are fewer boys than girls; if we run out of girls first, there are more boys than girls; if everybody has a partner, then there are as many boys as girls, and the set of boys is equivalent to the set of girls. Notice that counting is not necessary.

We can describe this situation in slightly different terms. If the pairing off "comes out even" (all members of both sets have been paired off), then we say that the set of pairs establishes a one-to-one correspondence between the sets. If there are members of the second set left over, then we have a one-to-one correspondence between the first set and a proper subset (part, but not all) of the second. If there are members of the first set left over, then we have a one-to-one correspondence between a proper subset of the first set and all of the second set.

These manipulations with sets are intended to establish the concept of number. Children frequently come to the first grade able to "count" to a hundred or more, but they may have failed completely to connect the words for the numbers with sets of the appropriate size. Our concept of number is just this: The number of members of a set, or just the number of a set, is that property common to all sets equivalent to it. To us, four is the property common to all sets equivalent to:



or to:



We see from the pictures above that the set of cats is equivalent to the set of mice--it is easy to set up a one-to-one correspondence between these sets. However, these two sets are not equal. A set of cats is a very different thing from a set of mice. The number of cats is equal (not equivalent) to the number of mice.

The notion of equivalence thus underlies our concept of number. We will see that the notions of "more than" and "less than" lead us in a natural way to the idea of inequality of number.

After the vocabulary and the necessary notions of set manipulation have been established we begin the process of assigning spoken names to the numbers. (We sometimes call these oral numerals.) This proceeds roughly as follows: We first learn by sheer memory (some rote drill, at last!) the spoken words for 1, 2, ... , 10. We then use this sequence of words--which at first is really a sequence of nonsense syllables--by touching successively the members of a set, saying the words in succession until every member of the set has been touched exactly once. The last word spoken names the number of the set. Later we replace touching by pointing or just looking, and we think of the names rather than saying them. This entire procedure is called counting, and is considerably easier to learn than to explain. The mathematical foundation is simple. Thus, for instance, 8 is the number of members in the set {1,2,3,4,5,6,7,8}.

It is essential that the child connect the spoken words for numbers with the corresponding sets. We begin to establish this connection when arranging sets in order. Sets can be arranged in order using the notion of "more than" and the numbers of these sets is then in the natural order--the same order as that in which we have spoken the words for 1, 2, ... , 10. We develop skill in counting by counting many sets, and we hope that, at this stage, the students will learn to count sets of at most ten objects with relatively few errors. For the smaller numbers, 0, 1, 2, 3, 4, and 5, we hope for a stronger identification. With enough practice children can preceive without counting the number of members in a set having no more than five members. We try to build this perceptive ability by many examples of such sets arranged in different sorts of patterns. (Our introductory unit on addition and subtraction presupposes quick recognition of the number of members of sets having at most five members.)

There are two important facts which children should discover in connection with counting. First, the order in which a set is counted is immaterial. Another child counting the same set but starting with a different member and proceeding in a different order will get the same number. Second, if two sets are equivalent, counting will yield the same result. This last fact shows us that the name we assign by counting a set really depends only on the number of the set.

In conclusion we may notice (but not necessarily point out to the children) that, essentially, we never count beyond ten. Because of our very clever system of numeration we can name the number of members of any set by repeatedly counting sets of ten.

III-1. Pairing and equivalence

Objectives: To introduce the idea of pairing members of sets
To introduce the idea of equivalence of sets

Vocabulary: Pair, equivalent, as many as

Materials: 9" × 12" paper, crayons, name cards, flannel board materials, individual set materials

Suggested Procedure:

Have name cards for only those children present today so that sets of cards and children are equivalent.

> I have a set of name cards for the set of children here. I have removed the set of cards of the children who are absent. What cards do I have now? (The set of name cards of the children here.)

Ask the children to suggest ways that the cards might be passed out.

> As I hold up your card, please come, get the card, and put it on your desk.
>
> We say that each member of the set of cards is paired with a member of the set of children.
>
> Is each name card paired with a child? (Yes) Is each child paired with a name card? (Yes)
>
> When members of one set are paired with members of another set and there are no members left over, we say that there are just as many members in the first set as there are in the second set. We call these equivalent sets.

At the top of the flannel board put a set consisting of two bananas, two apples, an orange and a lemon; also, another set of geometric shapes consisting of two circles, two triangles and two squares. Have the two sets described.

> Let's see if we have equivalent sets of fruit and geometric shapes. What can we do to find out? (Pair the fruit with the shapes.)
>
> Alice, will you please start the pairing using one member of each set? Choose any geometric shape and any kind of fruit that you would like.

Continue this process until the sets are paired. Stress the fact that it makes no difference which member of one set you choose to pair with a member of the other set.

> Are the sets equivalent? (Yes)

On the flannel board, place a set of five trees and directly below a set of seven circles so that the first five are obviously paired. Have the sets described.

> Is the set of trees equivalent to the set of circles? (No)
>
> What can you do to make a set of trees which is equivalent to the set of circles? (Supply more trees to pair with the circles.)

At the same time or maybe a day or two later, place a set of six chicks and a set of seven ducks on the flannel board so that the pairing of the first six is easy to see. Again have the sets described.

> Is the set of chicks equivalent to the set of ducks? (No)
>
> What can you do to make the set of ducks equivalent to the set of chicks? (Take away the duck that isn't paired with a chick.)

Each child will need six or seven set objects on his desk. The teacher will need an equal number of objects for the flannel board.

> Let's see if you can pair the members of a set on your desk with the members of a set I put on the flannel board.

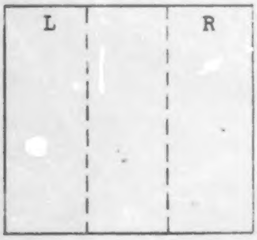
The teacher will put one object (rabbit) on the flannel board. Ask the children to put their equivalent sets in the middle of their desks so that they can be seen quickly by the teacher.

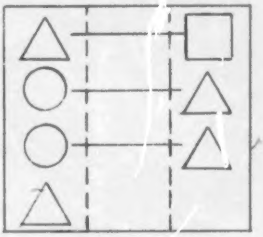
> I will put another member with my set. Now I have a new set. Is your set equivalent to my new set? What will you need to do to your set?

Remove the objects and start again with a set of two rabbits. This time remove a member of your set and continue as before. Use no more than two or three members in your set so that the child can see it without counting.

Distribute 9" × 12" papers that have been folded into thirds. Have each child make a picture of a set on the left third of his paper and fold that third so that his work does not show. The paper is then passed to another child who makes a set on the right third without looking at the other set.

The illustrations can then be collected and used for group work in finding sets which are equivalent. If desirable, they could be marked.





Pupil Page 7: Draw lines to show how members of two sets can be paired.

Pupil Page 8: Draw a set on the right equivalent to the set on the left. Encourage children to make simple pictures and pictures of different objects in each set if they wish to do so.

Further Activities:

Many classroom situations offer examples of pairing: the pairing of chairs and children, the pairing of books and children in the reading circle, the pairing of children for singing games, for relays, etc.

Flannel board demonstrations of pairing can also be used. Here it might be well to use some sets that are too large to count quickly.

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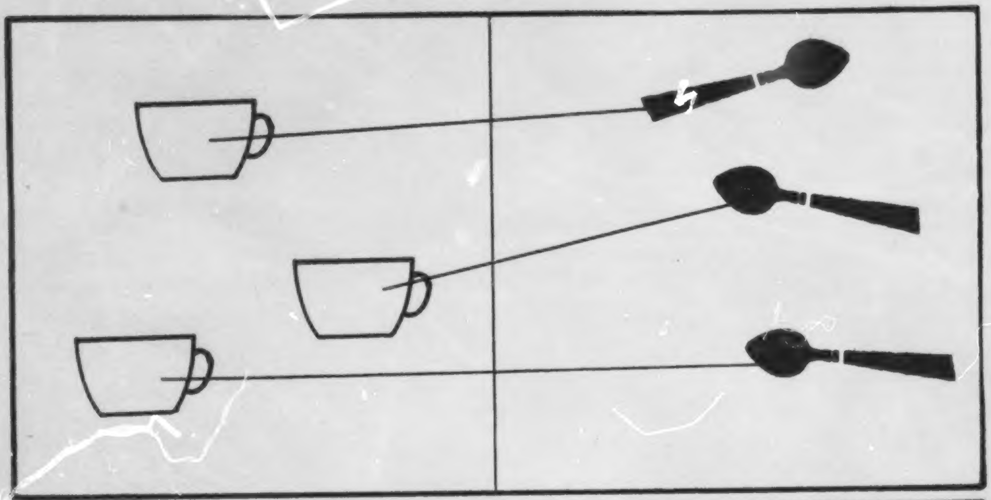
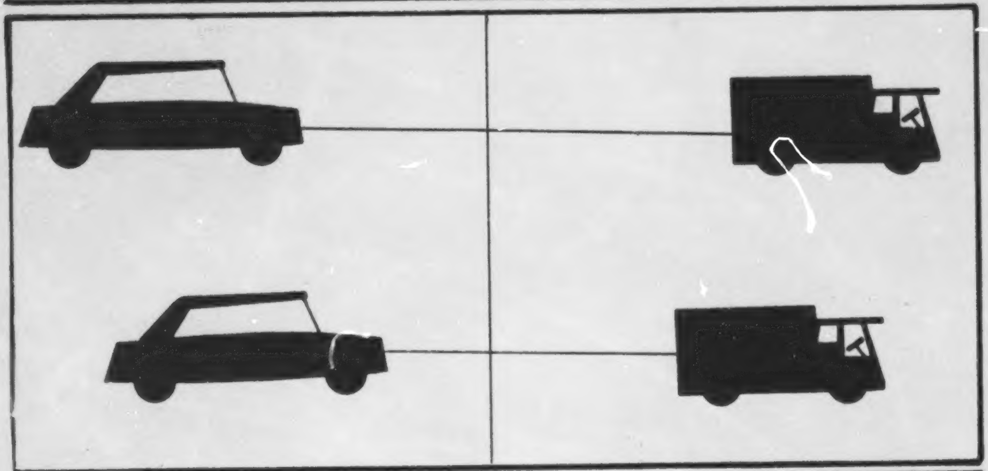
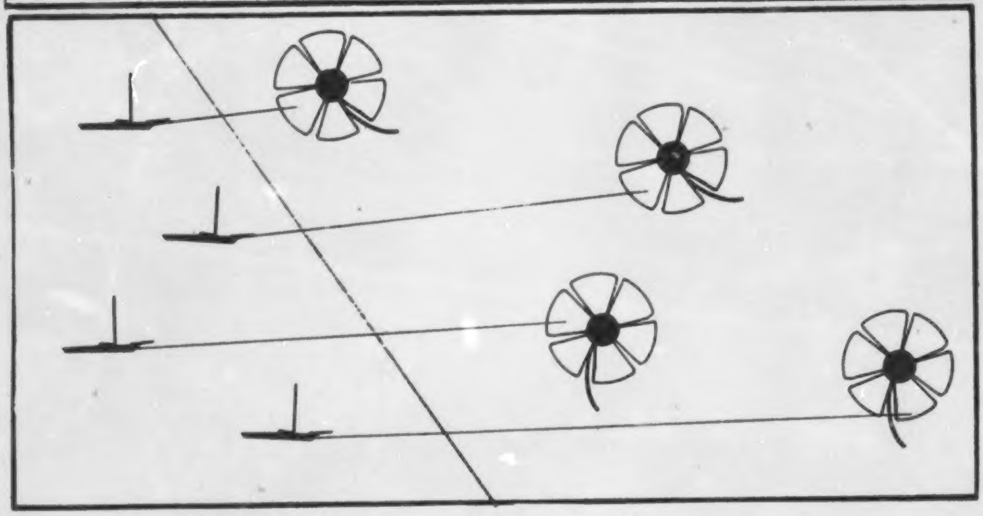
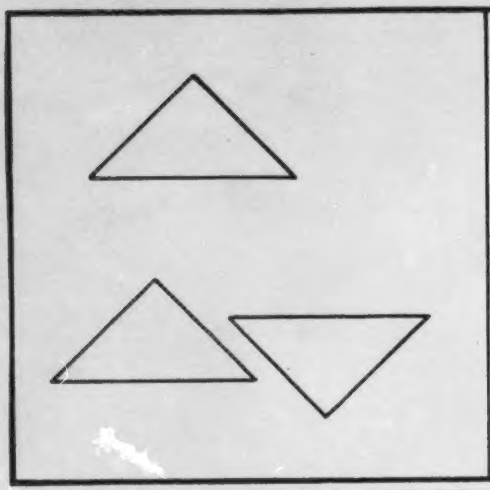
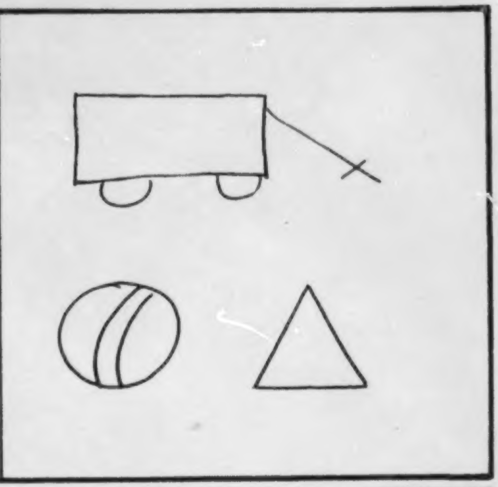
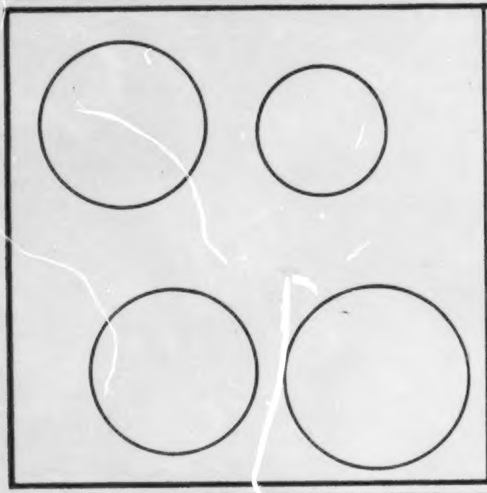


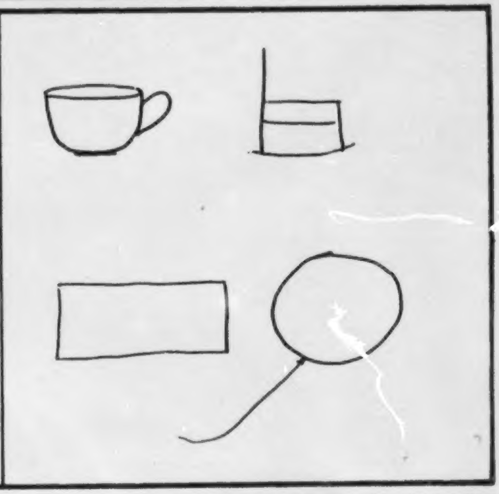


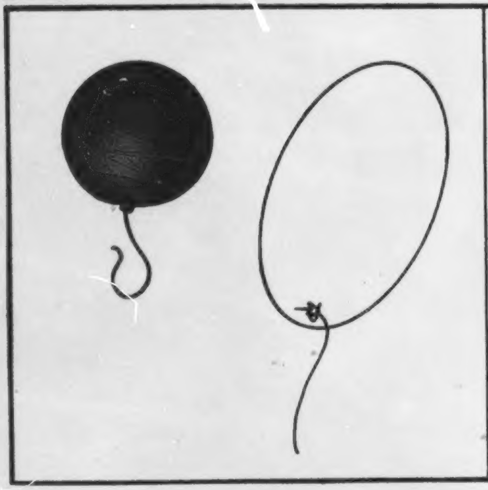


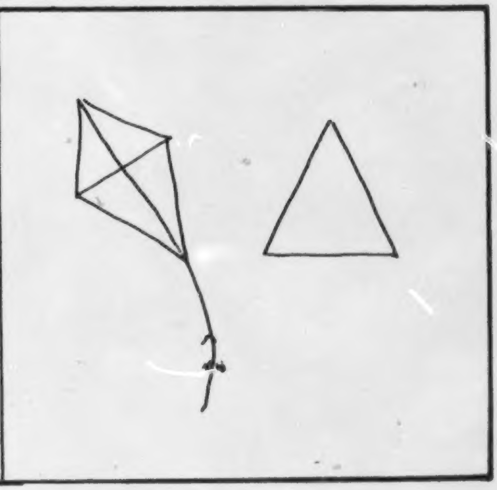












III-2. Comparison of sets

Objective: To introduce the idea that a set may have more or fewer members than another set

Vocabulary: More than, fewer than

Materials: Colored paper or felt circular and triangular cutouts or other materials from individual set objects, pictures of four story book characters (Goldilocks and the Three Bears)

Suggested Procedure:

Give each child a handful of circular and triangular cutouts or have these already placed in the individual set boxes or envelopes. Include a greater number of circular ones. Have the children describe the sets and ask that the members of the sets be paired on the desk. Offer no help except when the pairing idea is misunderstood.

> Were you able to pair all the members of your set of circles with all the members of your set of triangles? (No, there were some circles left over.) Yes, we say that your set of circles has more members than your set of triangles and that your set of triangles has fewer members than your set of circles. Let's look around our room. Can you see a set that has more members than another set? Fewer members than another set?

Pupil Page 9: In the boxes on the right draw a picture of a set that has more members than has the set in the box on the left.

Pupil Page 10: In the box at the right, draw a picture of a set with fewer members than has the set in the box on the left.

Further Activities:

Ask three children to stand by the flannel board. Ask another child to give a story character picture to each of the three children.

Is there a child for each character in the story? (No)

Are there more children than there are story characters? (No)

Are there as many children as there are story characters? (No)

Are there fewer children than story characters? (Yes)

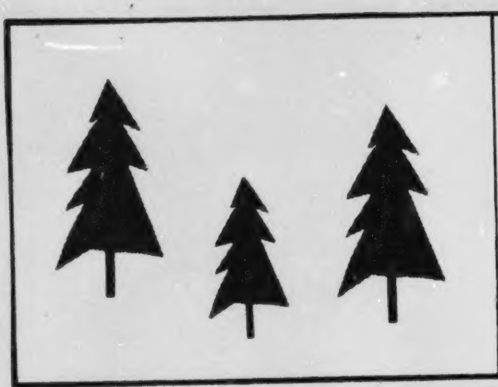
Ask the three children to put the pictures back on the flannel board and have another child come up to pass out the pictures. Encourage this child to pair the pictures and the children in a different order.

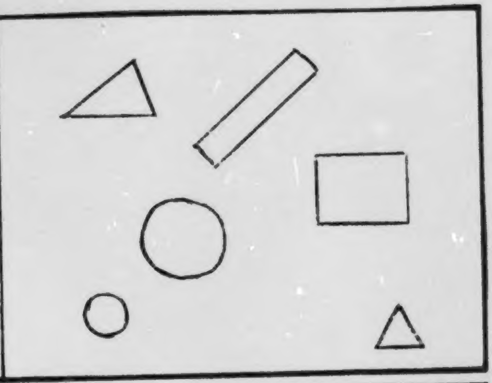
Do we now have as many children as story book characters? (No)

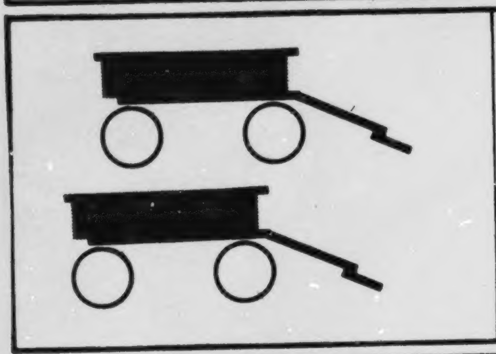
Are there fewer children than story characters? (Yes)

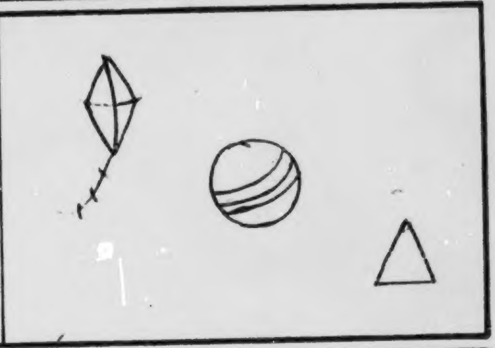
How many more story characters than children do we have? (One)

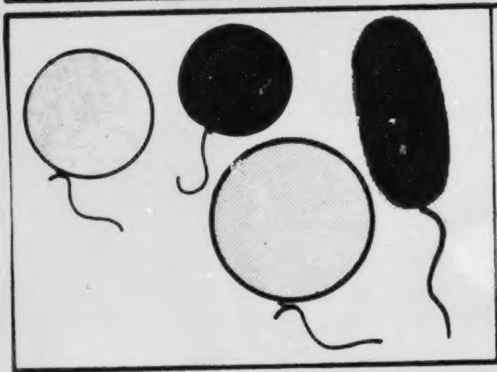
How can we make a set of children which is equivalent to the set of story characters? (Supply one more child to be paired with the one remaining story character.)

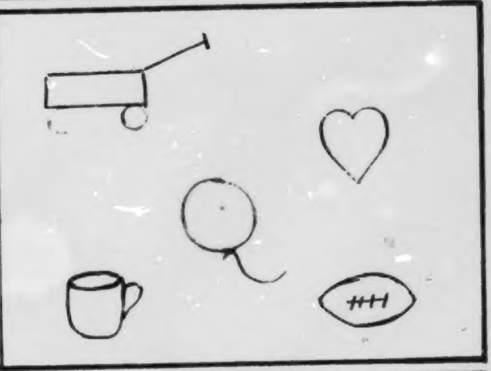


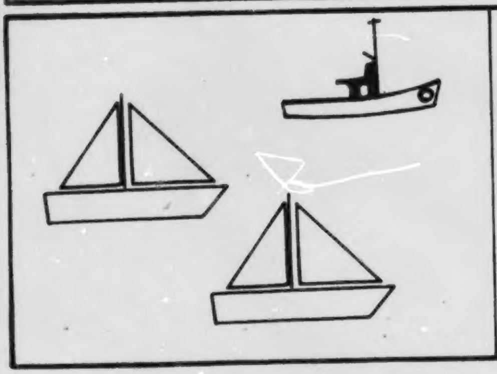


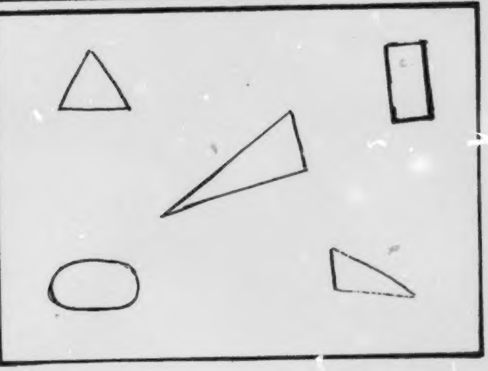


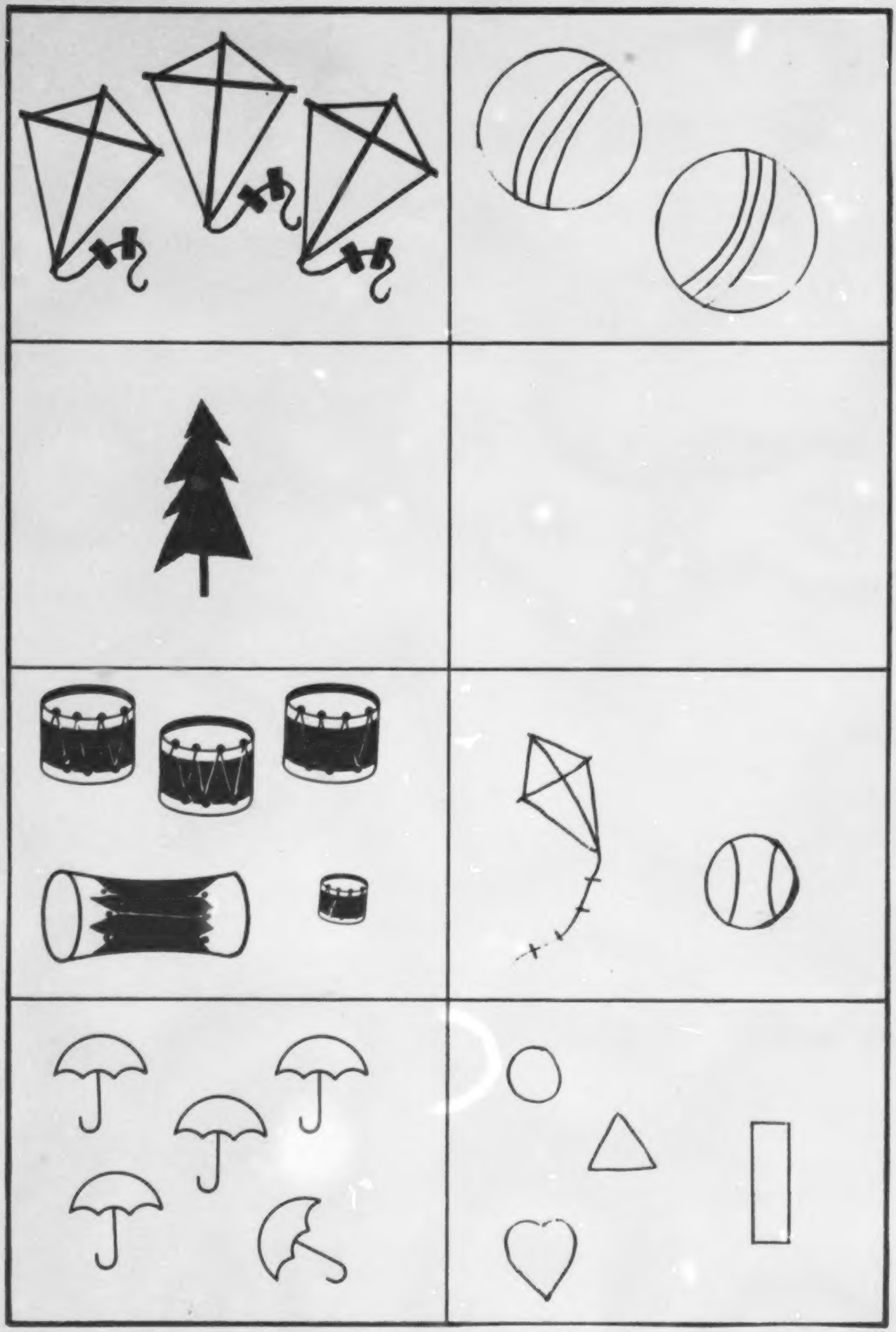












III-3. Review

Objectives: To review the concepts: equivalent, more than, fewer than, as many as, pair

Vocabulary: No new terms

Materials: Flannel board materials

Suggested Procedure:

On a desk by the door, place as many felt trees as there are boys in the class and as many felt ducks as there are girls in the class. As the children enter the room, ask each boy to take a tree and put it on his desk and each girl to take a duck and put it on her desk.

> We discovered the other day when we paired our set of name cards with the set of children in our room that we had just as many members in the first set as we had in the second set. Do you remember what we said about these sets? (Yes: equivalent) We also discovered that two sets aren't always equivalent. We know that a set may have more members than another set or fewer members than that set. Let's check today to see if our set of boys is equivalent to the set of girls. If the two sets are not equivalent, then we can say that the set of boys has more or fewer members than the set of girls.

Have the children pair the members of the two sets (trees and ducks) by asking a boy and a girl to go to the flannel board together and place their materials side by side. Continue until all children have gone to the board. In case there are more boys or more girls, they won't all be able to go in pairs, of course.

> We have said that in equivalent sets there are just as many members in one set as there are in the other. Is the set of boys equivalent to the set of girls?

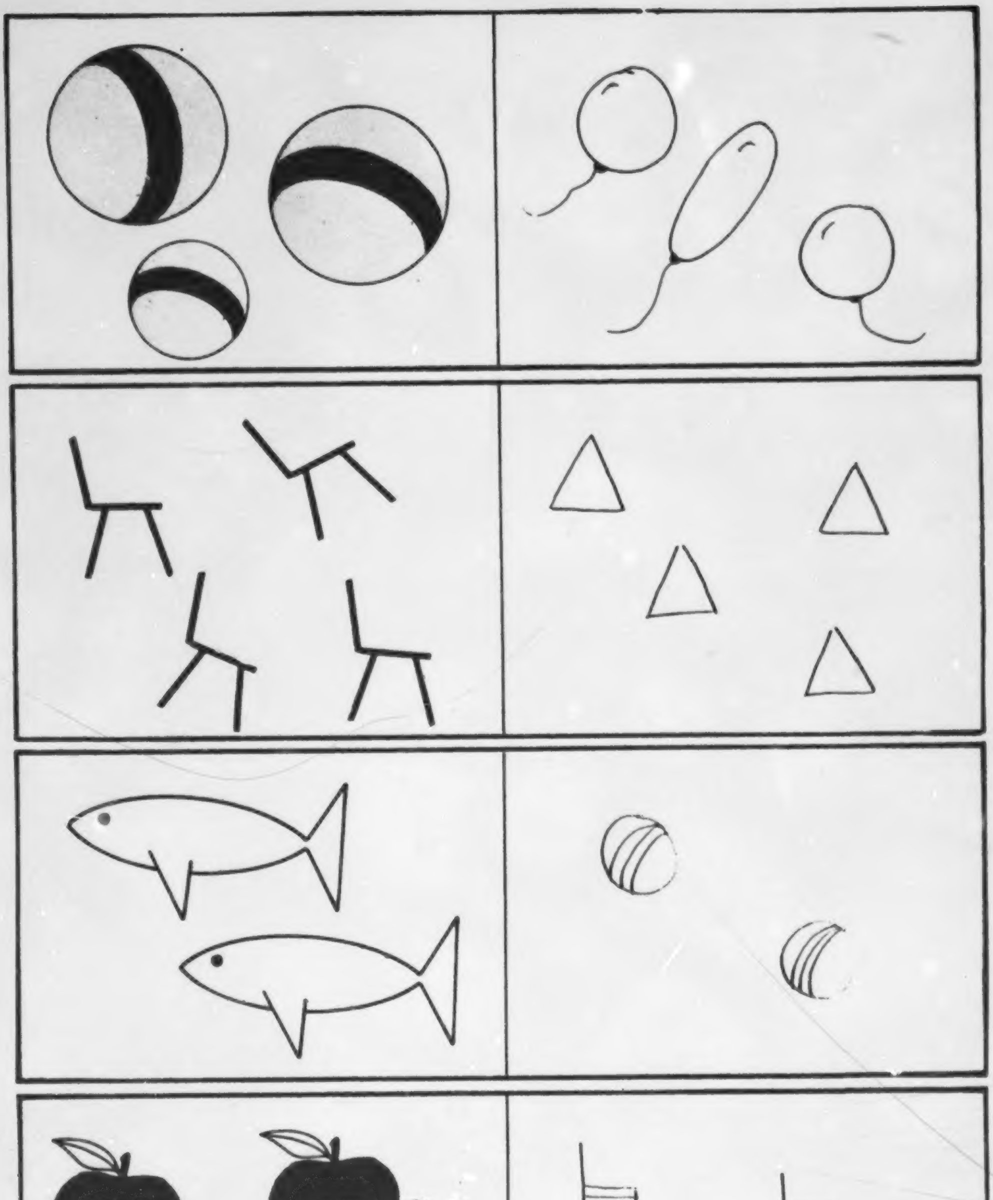
If the answer is "No," continue with these questions.

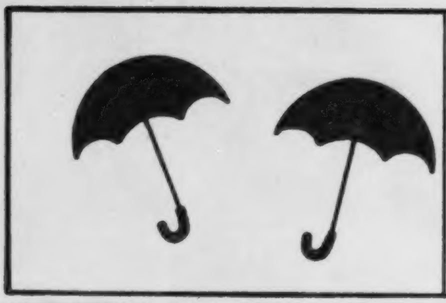
> Can we say that one of our sets has more members than the other set?
>
> Which set has more members?
> How can you tell?
>
> Can we say that one set has fewer members than the other set?
>
> Which set has fewer members?
> How can you tell?

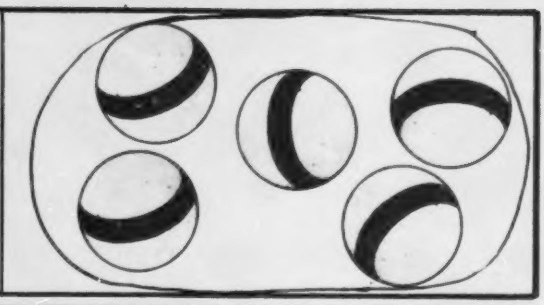
Pupil Page 11: Make the set on the right equivalent to the set on the left.

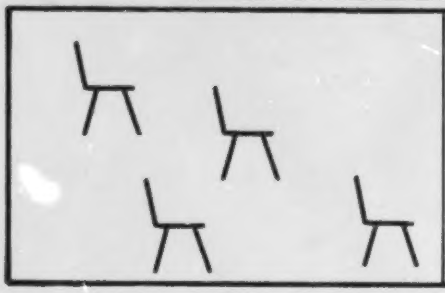
Pupil Page 12: Draw a ring around the set that has more members.

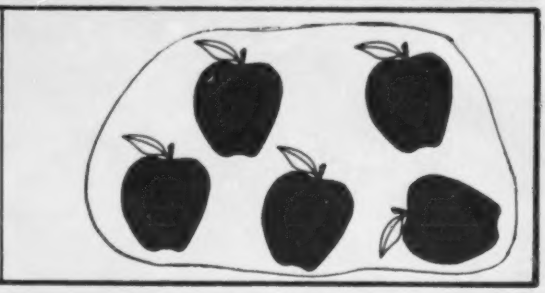
NOTES

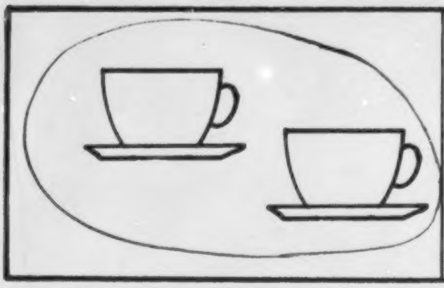


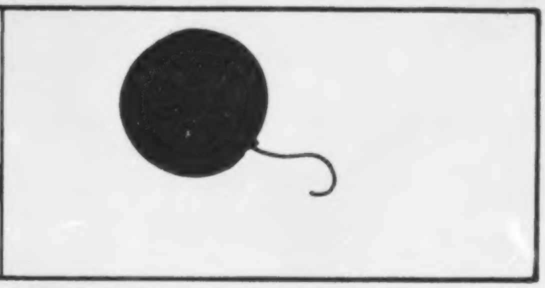


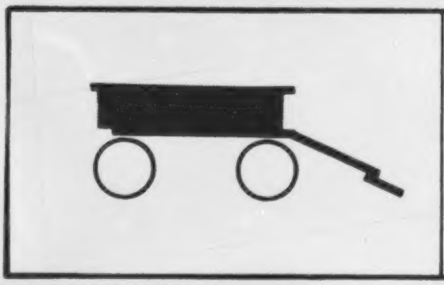


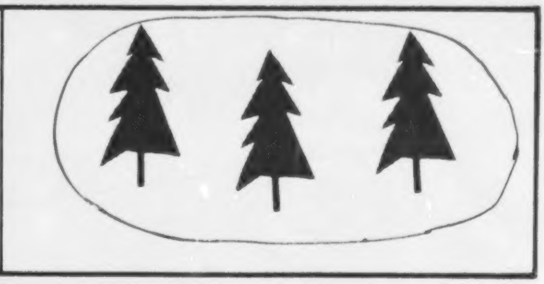


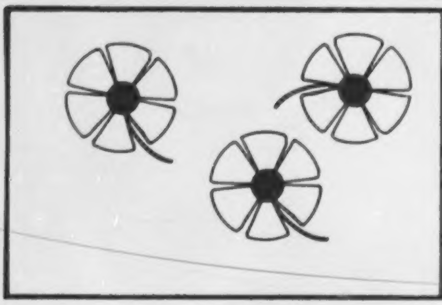


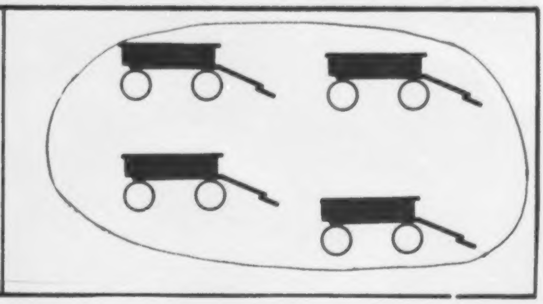












III-4. Comparison using imagined sets

Objective: To deepen understanding of comparison of sets by using imagined sets in comparisons

Vocabulary: More than, fewer than, equivalent

Materials: A set of three objects (e.g., book, pencil, toy car); a set of three objects for the flannel board

Suggested Procedure:

Place the three objects on the table and have the set described.

> Let's all imagine the set whose members are the Gingerbread Boy and the Little Old Man and the Little Old Woman. And let's pretend that we pair the Gingerbread Boy with this book, the Little Old Man with the pencil, and the Little Old Woman with the toy car. Do we have any story characters left over? (No) Do we have any objects on the table left over? (No) What can we say about the sets? (They are equivalent.)

Place three objects on the flannel board.

> We are going to pair the members of this set (indicate flannel board objects) with a set we can imagine. Let's think about the Big Billy Goat Gruff, the Middlesized Billy Goat Gruff, and Little Billy Goat Gruff, and the Troll.

Lead the children to talk about how the pairing might be done. Establish the fact that since there is always one story book character left over, the set we imagine has more members than the set on the flannel board.

Further Activities:

Repeat using various imagined sets, sometimes comparing imagined sets with each other rather than with sets of physical objects. It is probably wise to confine these activities to sets of four or fewer members, as children tend to find larger imagined sets difficult to keep in mind.

NOTES

III-5. Order and number names

Objectives: To introduce the idea that a collection of sets can be arranged in order

To introduce the spoken names of the numbers zero through ten

Note: In most classrooms the spoken names for the numbers will have been used many times in many connections before this lesson occurs, and of course this usage is to be encouraged. In spite of this early experience the children may fail to connect the names with sets of the correct number; and they may not realize that the number names are ordered in the usual way because collections of sets are ordered by the notion of "more than" (that is, that "five" comes after "three" because a set of five is more than a set of three). In this, the first lesson in which number names appear, this situation is clarified for the children.

Vocabulary: Arrange in order

Materials: Flannel board; objects to make a number of sets; student set boxes with objects

Suggested Procedure:

Place on the flannel board, at the top, a set of five objects. Ask each child to take enough objects from his set box to make an equivalent set and to place the objects at the top of his desk. Check to give individual help as needed. Place a set with two members in the middle of the flannel board and ask each child to make in the middle of his desk a set that has as many members as the set on the board. Place a set with three members at the bottom and ask the children to follow the same procedure at their desks.

> Which set on the flannel board has the fewest members? (The set in the middle)
>
> How could we change the sets around so the set with the fewest members would be at the top? (Place the set with two members above the set with five members.)

> Who will tell us something about the set in the middle? (Bring out that it has more members than the set at the top and the set at the bottom.)
>
> Can we change the sets around so that the set with the most members is at the bottom? (Interchange the set of five and the set of three.)

After getting the sets arranged with the set of two at the top, the set of three below it, and the set of five at the bottom, go on to introduce the term "order."

> How is the set at the bottom different from the set at the top? Look carefully at the sets before you answer. (The set at the bottom has more members.) If each set as we move down has more members than the set just before it we can say the sets are in order.
> Why are these sets in order? (Each set has more members than the set just above it

Next work toward placing other sets on the flannel board, always being careful to keep the sets in order. For example, you may attempt to put a set of three between the set of three which is on the board, and the set of five.

> Are these sets in order? (No, because the set next to the bottom does not have more members than the set above it.)

Keep working with the class until you have arranged in order on the board, and each child has arranged at his desk, sets of one, two, three, four and five members (or possibly up to ten members). The children should be sure that they cannot put a set between any two of the sets and still have the sets arranged in order. Bring out that the empty set belongs at the very top of the flannel board.

Associate numbers with sets of objects.

> Now we are going to talk about numbers. How many members are in this set? (Indicate a specific set.)

Then point a different set.

> What is the number of this set?

Use questions involving "how many members," "number of this set," and "number of members in the set" interchangeably until the numbers of all of the sets have been identified. Supply any number names not known by the children. Then reinforce the connections by pointing at various sets at random and asking for the number. Continue by trying to clarify the relation between arranging sets in order and the order in which the number names occur.

> John, come up to the flannel board and point to the set with five members. Now point to the set with two members. Which set is above the other? (Set of two) If we name these numbers, which name comes first? (Two)
>
> Mary, if you are naming numbers, does four come before five? (Yes) If we arrange a set of four and a set of five on the flannel board is the set of five above the set of four? (No, the set of four is above.)
>
> Can we put a set in between the set of four and the set of five? (No) Is there a number that comes between the number four and the number five? (No)
>
> What did we call the set that comes before the set with just one member? (The empty set) What is the number of members in the empty set? (Zero)

Why do we sometimes say that five comes after three? (Because a set of five has more members than a set of three.)

This last question can be pursued in depth. Bring out that since a set of five is more than a set of three, this means that a set of three is equivalent to a proper subset of a set of five. Demonstrate by pairing the members of such sets.

Pupil Page 13: Tell the children they are to place a mark in the boxes where the sets are in order. Then suggest they use red crayon to show the triangles.

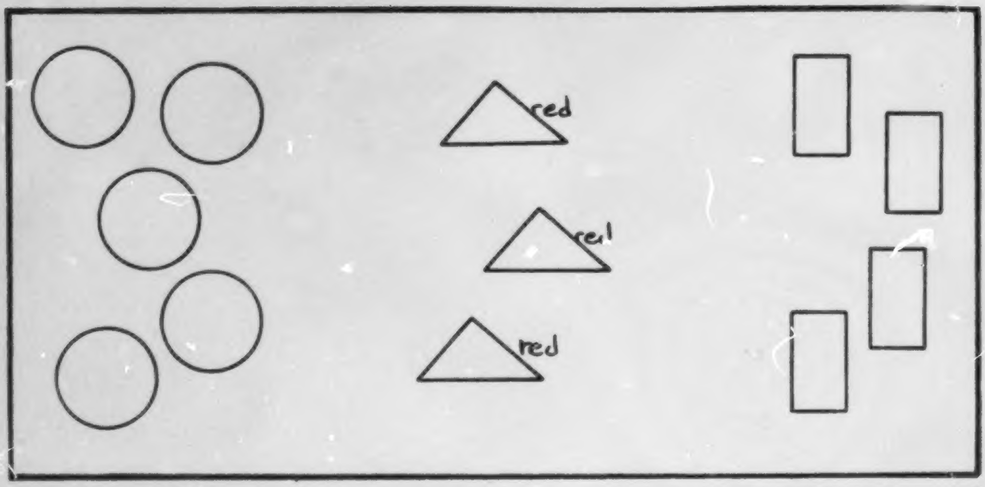
Pupil Page 14: Indicate the pictures of the sets are to be redrawn in order from top to bottom. On the chalkboard draw a triangle with red chalk, a rectangle with blue chalk. Ask the children to color the models on their sheets.

Further Activities:

Other lessons can be used as needed, using different small sets to be ordered. Keep reinforcing the connection between arranging sets in order and the order in which numbers are named.

Organize a book with sets of from one to five objects.

Read stories:
Slobodkin: One Is Good but Two Are Better
d' Aulaire: The Two Cars
Meeks: One Is the Engine
Watson: What Is One
Schlein: The Four Little Foxes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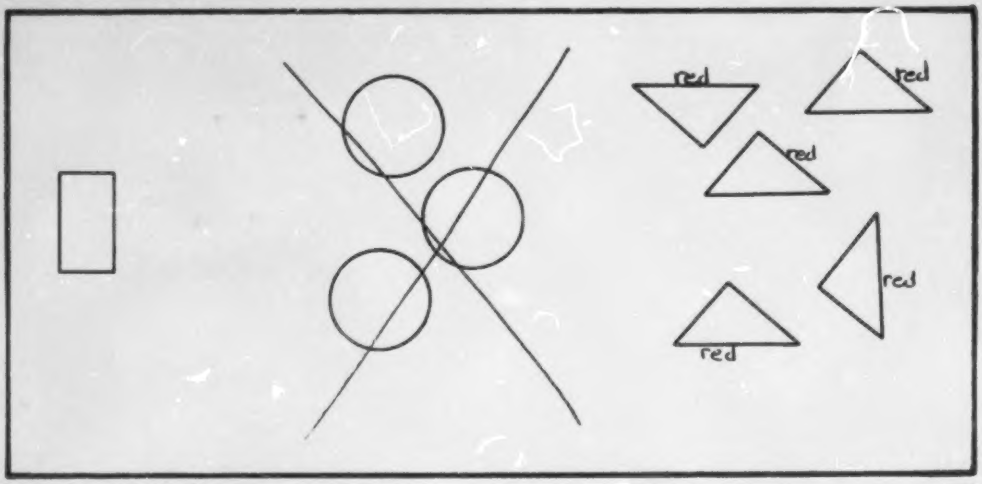
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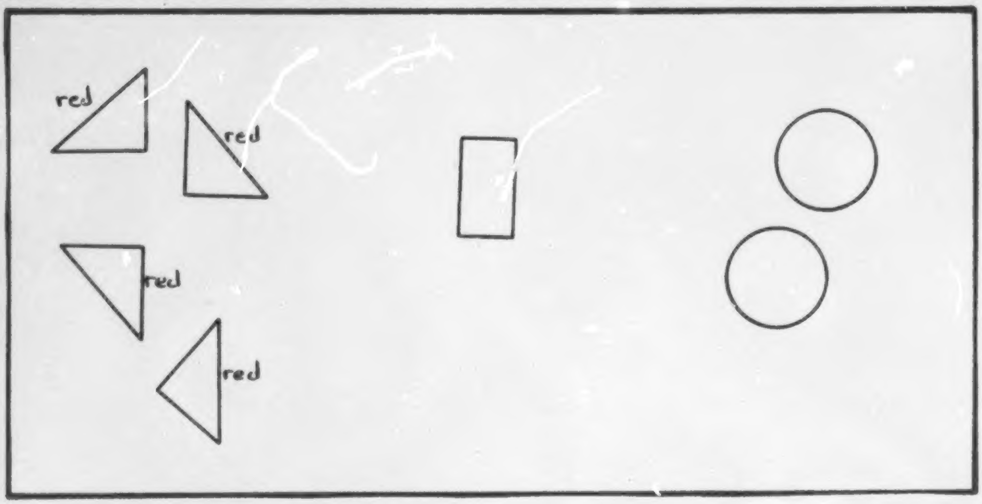
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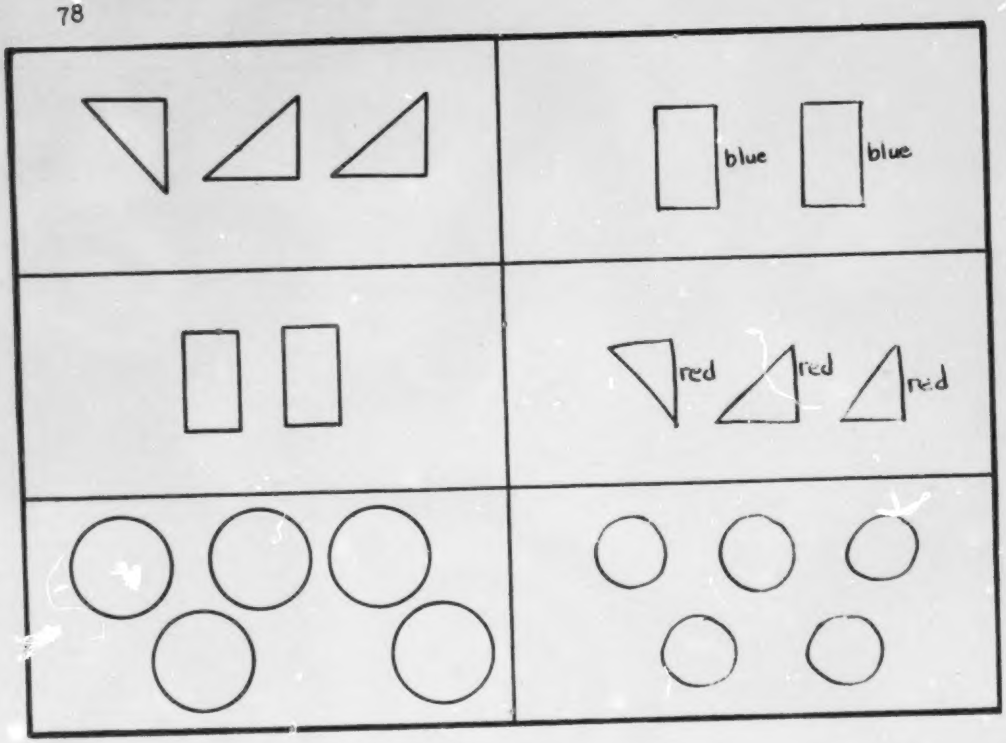
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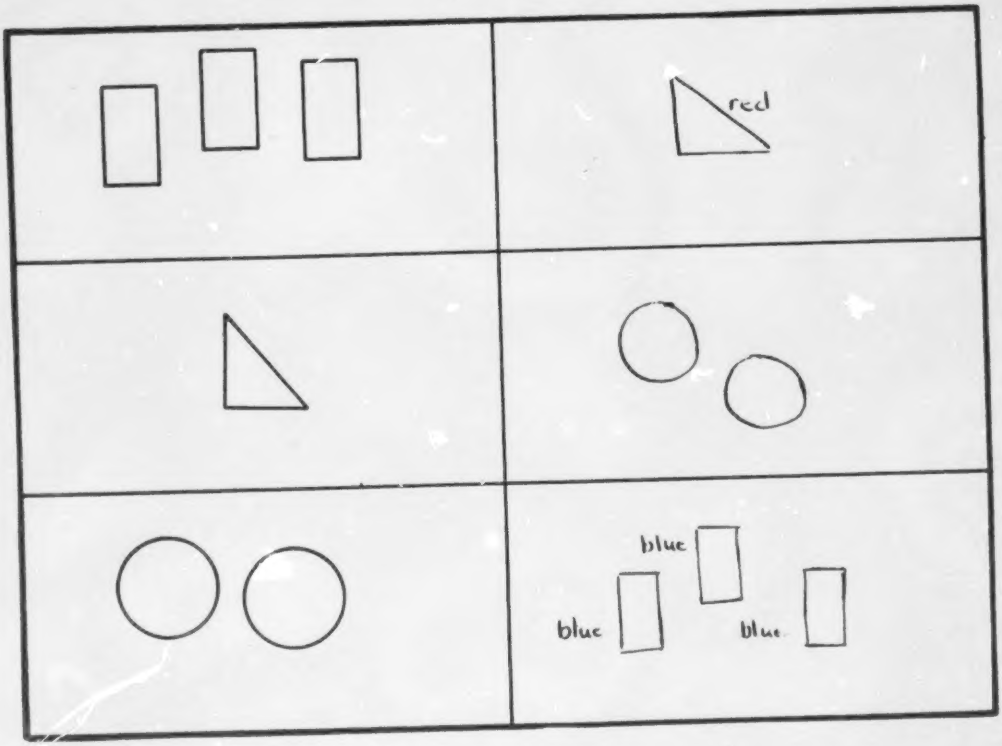
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III-6. The number zero

Objective: To reinforce the child's understanding of the number zero

Materials: A set of five objects (pencil, crayons, eraser)

Suggested Procedure:

Place the set of objects on a demonstration table. Have children grouped around where all can see. Call attention to the number of the set (five). Ask a child to come up and take an object to his chair. Repeat, each time asking for the number of the remainder set.

When the last object is removed, ask about the set now on the table.

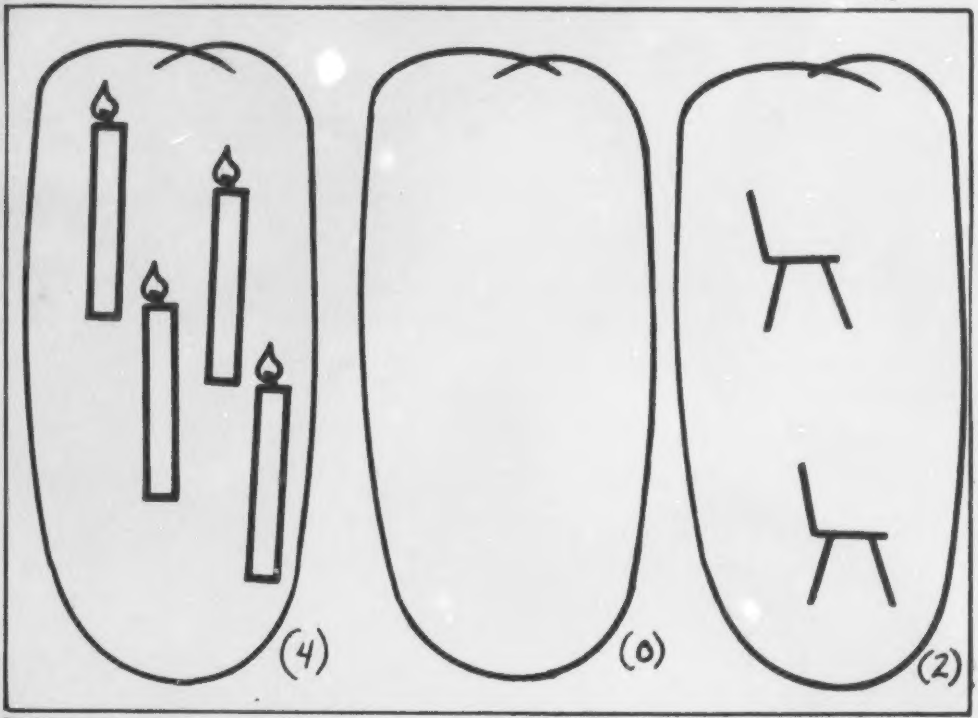
> What set do we have now on the table? (The empty set)
>
> Do you know a number that tells how many things there are in the empty set? (Zero)
>
> Zero is the number that tells us how many things there are in the empty set.

Continue with illustrative questions such as:

> What is the number of giraffes in our room today? (Zero)
>
> What is the number of boys in this room who have green hair? (Zero)

Count a set of objects and put them in a grocery bag. Remove one object at a time and ask a child to tell the number of objects still in the bag. Continue until there are no objects left in the bag. Again ask a child to give the number of objects in the bag. Reinforce the idea that zero is a number.

Pupil Page 15: Ask children to tell the number for each set. Indicate these sets are to be drawn in order in the box at the bottom.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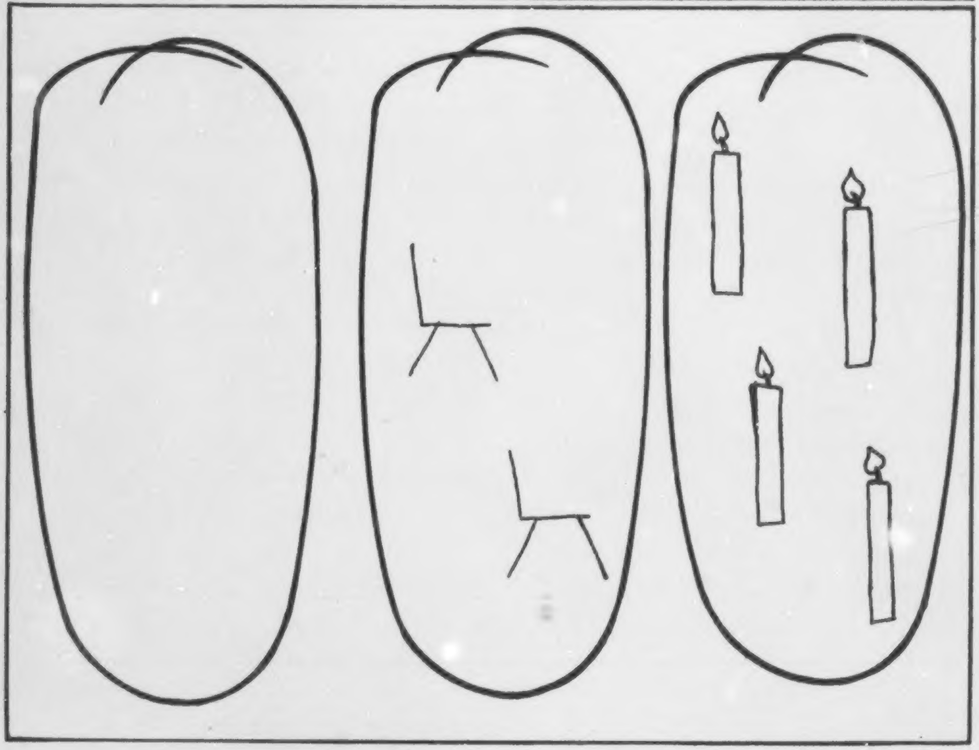


III-7. Counting objects

Objective: To help the child to learn to use counting ten members to identify the number of a set

Note: For many classes this lesson is entirely unnecessary. In any case, this is the appropriate time to check that children are able to count sets of at least five, and preferably ten, objects and to help those students who need further practice. The lesson below is given in abbreviated form because many classes will need little or no practice of this sort.

Vocabulary: No new terms

Materials: Sets of physical objects

Suggested Procedure:

The first step in learning to count is to learn by sheer memory the spoken names of the numbers from one to ten. There are many ways of aiding this memorization process. Nursery rhymes and songs such as "One little, two little, three little Indians," "One, two, three, four, five, I caught a hare alive" and "One, two, buckle my shoe" are very useful. At any rate, the child must learn to say the words in sequence. Of course, it is primarily a nonsense rhyme at the early stage, and even after attaining perfection in saying the words the child may have little conception of the meaning of, say, eight.

The next stage is the "touch and say" stage. The child touches each member of a set, saying in sequence the names of the numbers. The name spoken when the last member of the set is touched is the number of members in the set. We then proceed to the "point and say" stage, the stage of pointing and thinking, and finally to the "look and think" stage.

During the counting practice the child should be led to realize that the particular order of objects does not matter. That is, if another child counts the same set in a different way, beginning with a different member and touching the objects in different order then the same number of members is obtained. He should also be convinced that if two sets are equivalent then they have the same number of members, and that if two sets have the same number of members then they are equivalent. Thus, if a group of boys is counted and the same number of girls is counted, we are sure that we can form couples with no one left over.

This entire development is accomplished by a great deal of practice which should not be confined to the period set aside for the mathematics lesson. Children should count sets of chairs, sets of children, sets of toys, sets of desks, and so on as part of their everyday classroom activity.

NOTES

III-8. Number perception, without counting

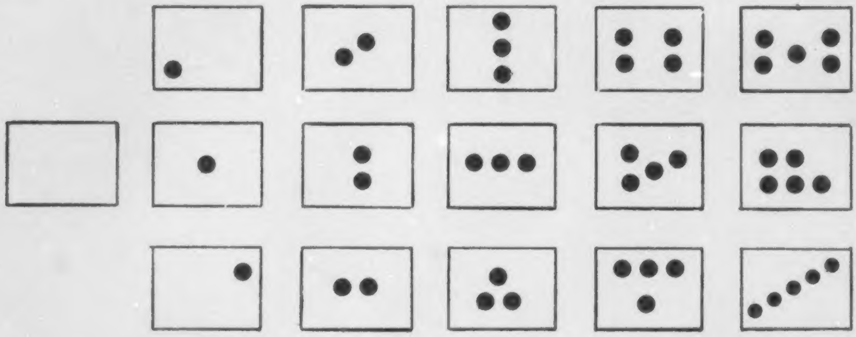
Objective: To help children perceive, without counting, the number of objects in a set of no more than five members

Vocabulary: No new terms

Materials: Set perception cards, in various patterns, for sets zero through five; flannel board materials

Suggested Procedure:

Use perception cards (without numerals) of sets of from zero through five members. Perception cards should have different arrangements of the members of the sets:



Give each child five objects. Show a card and have the child put on his desk a set equivalent to that displayed on the card.

Put a set of objects on the flannel board. Have children show equivalent sets at their desks.

Display a perception card very quickly; then conceal it and have children show equivalent sets at their desks.

Display a card, cover it, and have a child give orally the number of the set that was shown.

Further Activities:

There are many opportunities to point out equivalent sets in the classroom and to encourage children to perceive the number of the set without counting. For instance, if there are two sides of an easel, two children can paint. If there are four balls for the recess period, four children may take out balls. If there are three fish in a fish bowl, put three books about fish on the table beside them and encourage children to see "three," rather than "one, two, three." Encourage children to point out as many equivalent small sets as possible: one wagon, one clock, one piano, one teacher's desk, one coat rack, etc.

NOTES

III-9. Arranging sets in succession

Objective: To put sets in order so that each set has one more member than the set which precedes it

Vocabulary: Review: order, more than, fewer than

Materials: Five sets of objects (varied) with one to five members, flannel board, set materials for children, flannel pairing symbols ←——→ , or pieces of string

Suggested Procedure:

Place five sets of objects ranging from one to five members on the flannel board in some such set arrangement as sets of three, one, two, five, and four members. Use a top to bottom arrangement.

Ask how the sets can be put in order. Agree that the set with one member can be at the top.

> Does the set with three members belong next in order? (No)
>
> What is the number for the set that does belong next? (Two)

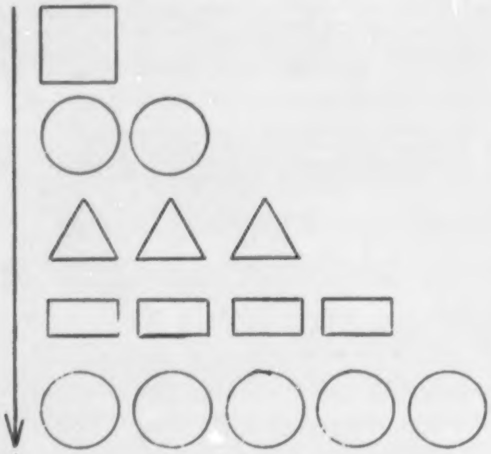
Ask for several ideas about a set with two members. Some desirable responses are the following:

> It has one more member than the set with one member.
> It has one fewer member than the set with three members.
>
> Two comes before three.
> Two comes after one.

With each of the above responses encourage the child to explain why he has given his response.

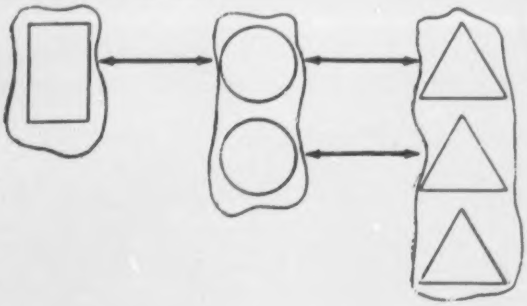
If one-to-one correspondence is clearly understood, the response can be expected that pairing the members of the set of one and two shows there is one more member in the set of two. This concept is basic in relating set order to number order.

Now complete the arrangement as shown, using techniques suggested above.



Take one set at a time and place in a left to right order as shown below. Ask appropriate questions as the sets are ordered.

Use a flannel pairing symbol or a piece of yarn between the members of the set of one and two members that can be paired, as illustrated. (To help keep in mind the three sets involved, they might each be delineated with yarn, as shown.)

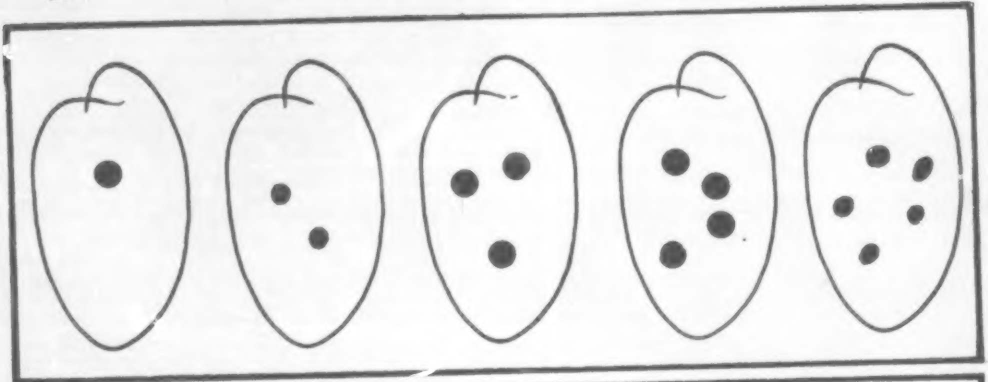


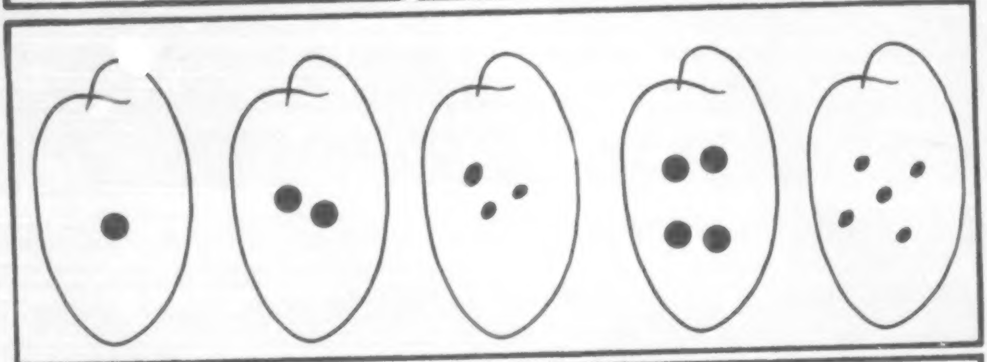
Cover the set with one member and the pairing symbol. Now ask about the sets of two and three. Follow the above procedure and continue placement of pairing symbols as illustrated. Proceed in the same way with the sets of four and five. When the process is completed, remove the covers to show the entire order.

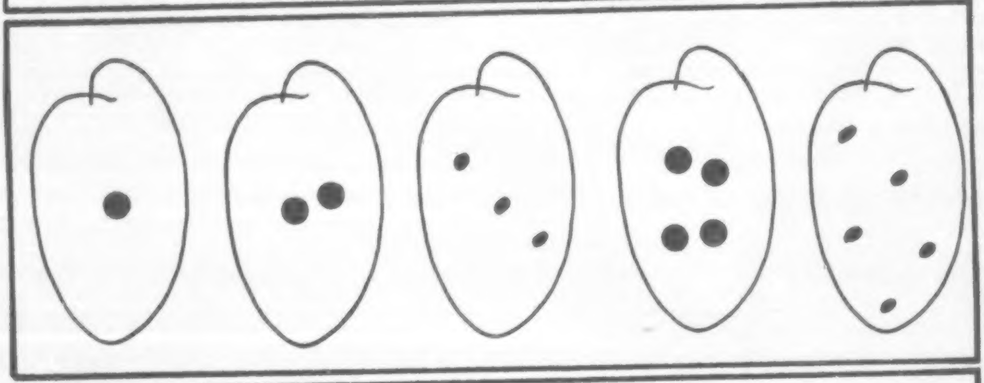
Remove the sets with two members and the set with four members and the pairing symbols. Ask for the number of each missing set and ask why this set belongs there.

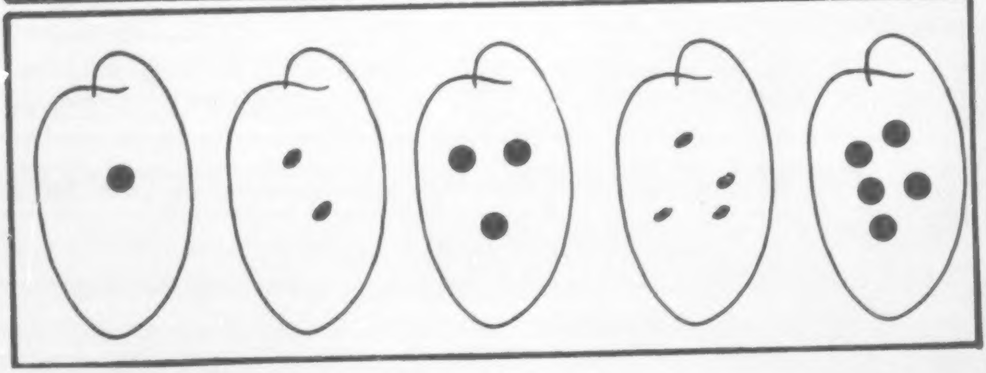
Pupil Page 16: Use this page for the idea of the set that is next in order. Missing sets are to be drawn.

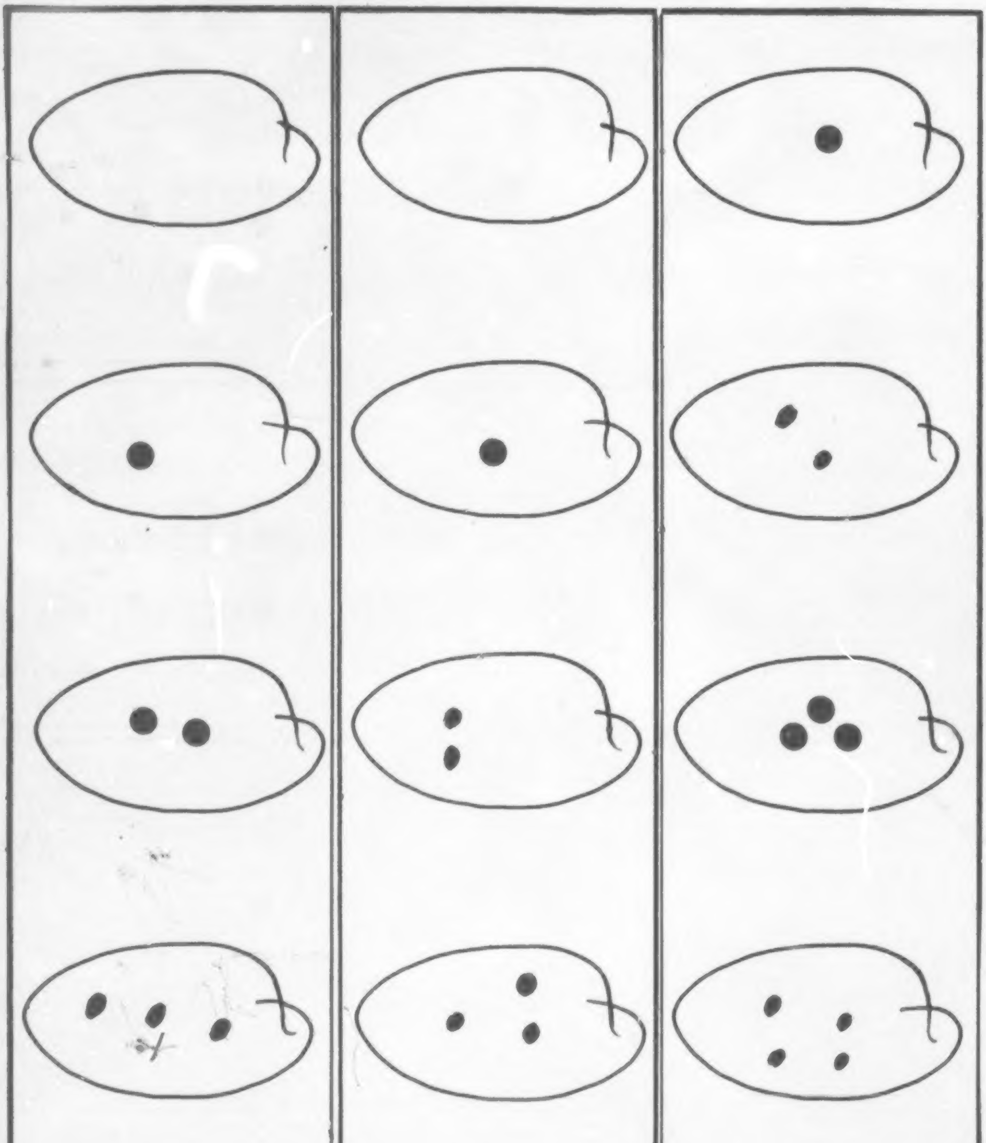
Pupil Page 17: Find several sets that are missing in order. These sets are to be ordered top to bottom. The empty set is included in the first two sets.











Chapter IV

PRE-ADDITION AND SUBTRACTION EXPERIENCES

Background

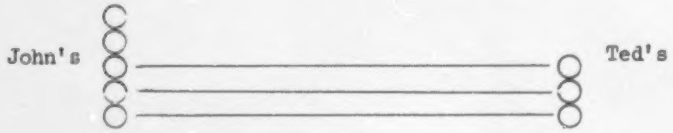
This chapter is concerned with manipulation of sets and identification of the number of members of the various sets constructed. This is preparation for later formal work on addition and subtraction. We use sets having at most 5 members in order to keep the mechanics of counting from obscuring the new ideas, and in order to reinforce the children's perception, without counting, of sets having very few members.

Addition. The notion of joining sets underlies the arithmetical notion of addition. Recall that the union of two sets consists of all objects which belong to one or to both of the sets. Thus, in a classroom the union of the set of all boys and the set of all girls is the set of all children. By the definition of plus, the number of boys plus the number of girls is equal to the number of children. Thus 2 + 3 is by definition the number of members in the union of a set of 2 and a set of 3. There is one complication (which you may or may not want to point out to your pupils at this stage.) The sets which we join must have no common members. (They must be disjoint.) Thus the set consisting of John and Mary has 2 members, the set consisting of Mary, Sue, and Jane has 3 members, the union of these two sets, which consists of John, Mary, Sue, and Jane, has 4 members, and 4 is not 2 + 3.

The operation of joining sets is commutative, in the sense that the set resulting from joining one set to another is the same as the set resulting from joining the first set to the latter. Because of this the operation of addition of numbers is also commutative so that, for example, the sum of 2 and 3 is equal to the sum of 3 and 2. Commutativity of addition is usually not obvious to the children and a special effort should be made to help them understand.

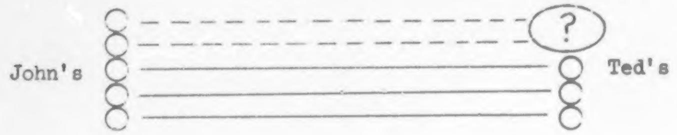
Subtraction. We will indicate by problems several of the ways to describe subtraction. The most commonly used definition of subtraction gives us the following sort of description: 5 - 3 is the number of members in the remainder set if a set of 3 is removed from a set of 5. Thus, if John has 5 marbles and his older brother takes away 3 marbles, then the remainder set has 5 - 3 members.

We may also consider the problem: if John has 5 marbles and Ted has 3 marbles, how many more marbles has John? We may think of pairing off Ted's marbles with John's, as shown below



removing from the set of John's marbles the set which is equivalent to Ted's, and counting the remainder set. Since the number of Ted's marbles is equal to the number of the equivalent subset of John's marbles, we see that John has 5 - 3 more marbles than Ted.

The following is a closely related problem: if John has 5 marbles and Ted has 3 marbles, how many marbles must we give Ted so that he has as many as John? Schematically, we can pose the question as follows:



If we remove from the set of John's marbles a subset which is equivalent to Ted's set, then the remainder set is equivalent to the unknown set. We conclude that we must give Ted 5 - 3 marbles.

This last description of subtraction in terms of sets leads to a formulation which does not depend on sets, but only on the notion of addition which we have introduced already. (Of course the definition of addition does depend on manipulation of sets.) Suppose again that John has 5 marbles and Ted has 3, and we wish to know how many marbles to give Ted to equalize matters. The union of the set we give and the set Ted has must be equivalent to the set of John's marbles. Hence the number of marbles we must give, which is 5 - 3, is the answer to the following question: 3 + ? = 5. In the same way, 4 - 2 is the answer to the question, 2 + ? = 4, and so on. This is sometimes called the missing addend description of subtraction. It is important that children work with this description as well as with the descriptions in terms of set manipulation since this will be the fundamental notion underlying the subtraction of numbers in later grades. In general, we try to give the children experience with all of the ways that subtraction problems arise.

Inverses. We emphasize the close relation between addition and subtraction in another fashion. If we join a set to another set and then remove it from the union, the remainder is just the original set. If we remove a subset from a set and then join it to the remainder set, then we again have the original set. We sometimes say that joining a set, and removing the same set, are inverse operations, in the sense that doing these in succession to any set always gives back the original set. This fact about manipulation of sets shows us something about addition and subtraction. If we add 2 to a number and then subtract 2, we have the original number. If we subtract 2 and then add 2, we again have the original number. Thus, adding 2 and subtracting 2 are inverse operations, in that doing these in succession to any number always gives the original number. Of course, adding 4 and subtracting 4 are also inverse operations, and so on.

Partitioning sets. We introduce the notion of partitioning sets in this chapter. This notion is used here primarily for reinforcement of the various number relationships but we shall later use partitioning into equivalent sets in place-value and in the introduction to division. Partitioning a set is just separating it into two disjoint subsets. (We shall later partition into more than two subsets.) Thus, we may partition the set consisting of Mildred, Jean, Stan, and Mary into the set consisting of Mildred, Jean, and Stan, and the set consisting of Mary. Partitioning is related to both joining and removing. For example, if we join the set consisting of Mildred, Jean, and Stan to the set consisting of Mary we have the original set. (We will later relate this to the number sentence: 1 + 3 = 4.) On the other hand, we may think of removing the set consisting of Mildred, Jean, and Stan, in which case the remainder set consists of Mary. This will later suggest to us the number sentence: 4 - 3 = 1.

NOTES

IV-1. Joining sets

Objective: To introduce the joining of sets in association with the spoken names of numbers 0 through 5

Vocabulary: Review: joining, numbers 0 - 5, member, set, subset, empty set, set with one member, union

Materials:
1. Flannel board and objects
2. Chalkboard
3. Perception cards (see instructions under Further Activities: both picture and geometric shapes)
4. Sets of physical objects and demonstration table
5. Individual set boxes or the like

Suggested Procedure:

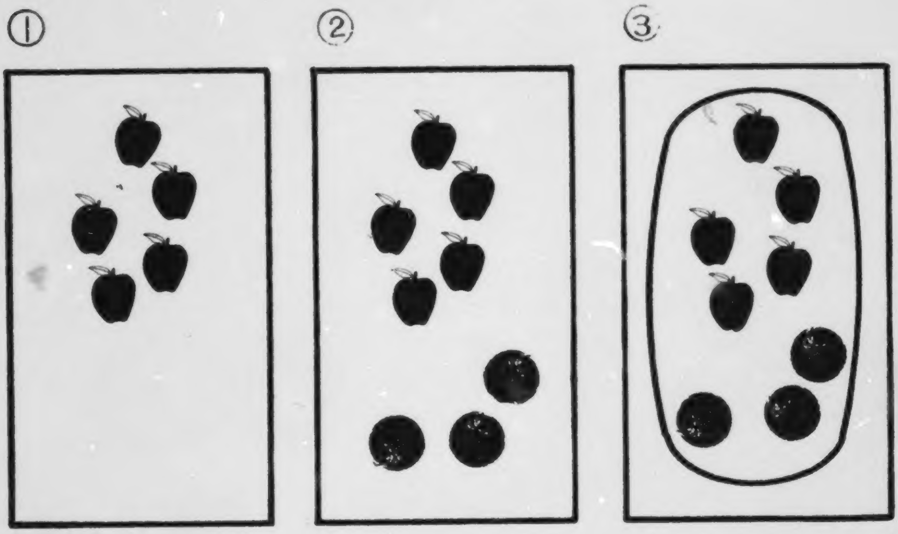
The previous experiences have introduced the children to joining sets by manipulation without using the spoken names of numbers. The class is also able to identify sets by counting the members. A combination of these two concepts will make the introduction of joining sets and associating the number orally with the resulting set interesting and easily understood. As a pre-addition activity the teacher will find it useful to establish an understanding of joining and counting both, the original and resulting set. Refer to the resulting set as the union of the sets joined. An example of introduction by means of the flannel board follows. (The same procedure is applicable to chalkboard illustrations with slight modifications as the teacher sees need.)

Place a set of objects on the flannel board and have the set described. (Example: set of apples, set of red apples, set of 5 apples, etc.)

Count the above set and place it to one side of the board. On the other side of the board place a new set. (Example: 3 oranges) Identify this set as you did the first.

> We now have a set of 5 apples on our board and a set of 3 oranges. Let's put the orange with the apple. What do we call this? (Joining) We began with a set of 5 apples and a set of 3 oranges. How many members do we have in the new set that we just made by joining the two sets? (8)
>
> The movement of placing one hand on one set and bringing it to the other clarifies the process.

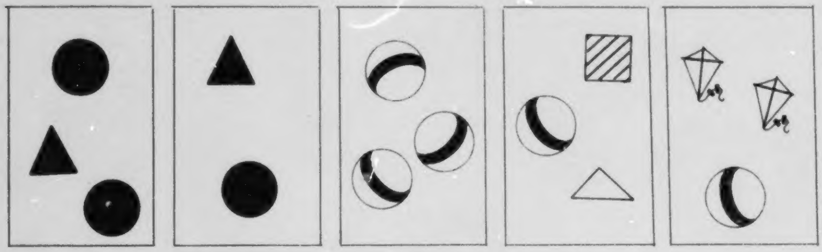
By moving the set on the flannel board, the children are able to see the joining of the separate sets and formation of the new set. Flannel board technique:



Pupil Pages 18 and 19: The sets at the left of the page are to be joined and the union is to be pictured on the right side. Some of these unions are correct and others incorrect due to deficient members or not the exact members as in the sets joined. (Note the first and second exercises on page 19.) Instruct the children to draw a ring around the word "yes" if the union is correct. If the union is incorrect, draw a ring around "no." Page 19 involves the more complicated activities.

Further Activities:

1. Perception cards as made by the teacher can help to visualize the joining of sets. A few examples of perception cards made on tag board with felt pen are shown below. Magazine pictures pasted on heavy paper are also useful. For example, a picture of a stack of pancakes may be pasted on one card and a picture of a bottle of syrup may be pasted on another card. Several advertisements are excellent examples and fun for the children to discover. They also enjoy making these cards themselves. Hold up one card and have the children count the members. Place this card on a stand and hold up another card using the same procedure. Join the two sets together and count the resulting set.



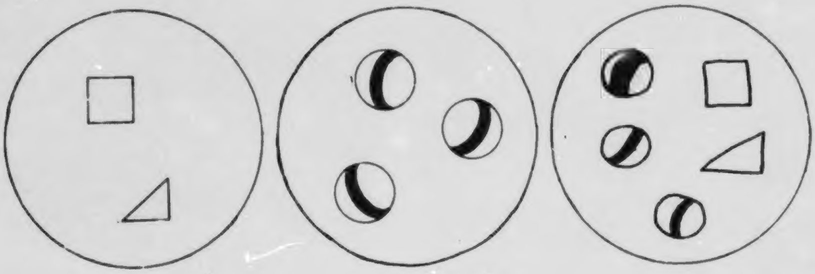
2. Sets of objects such as books, writing equipment, art supplies, blocks, etc. may be placed as individual sets on a large demonstration table. First identify two sets and join one to the other to form a new set. Remember to count the members of each set as well as the members of the union.

3. Individual set containers: boxes, envelopes, bags, etc. used at the pupils' desks to form the union of two sets. Note the materials section for suggestions. Proceed with the lesson in the same manner as discussed being certain to use orally the number in both original sets and in the resulting union.

4. Use the class members divided into two sets as you wish and count the sets. Join these sets and count the resulting set. Example: ball monitors ②, errand helper ①, union ③.

5. Chalkboard work such as:

Draw a set of 2 members }
Draw a set of 3 members } Have the children draw a picture of the union of these sets.



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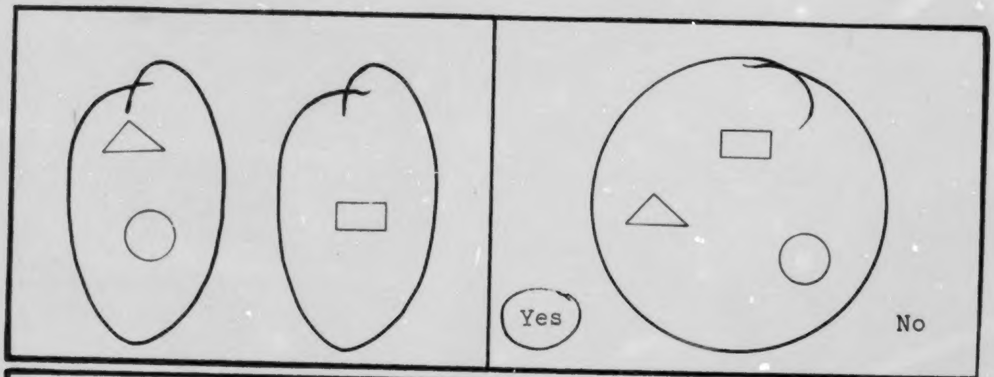

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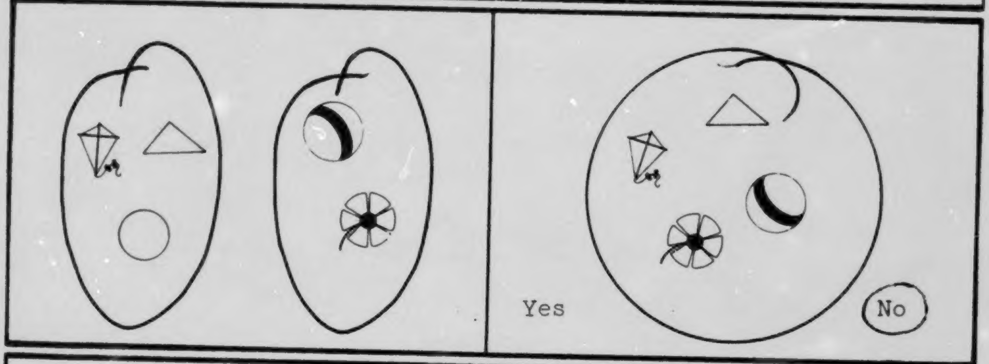

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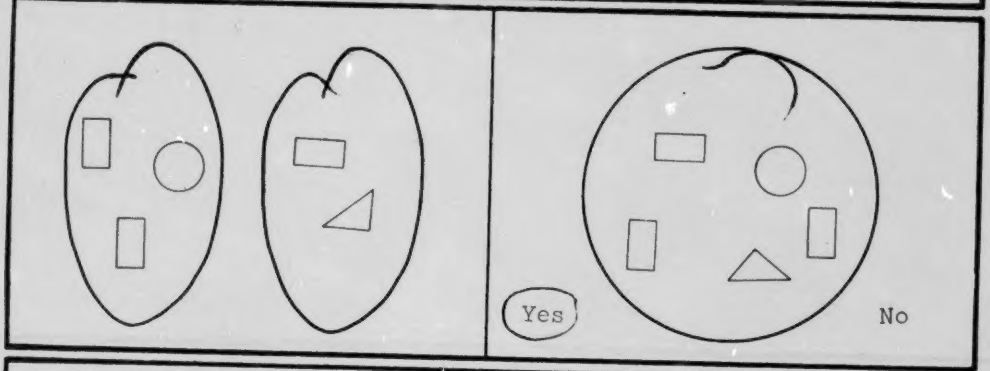

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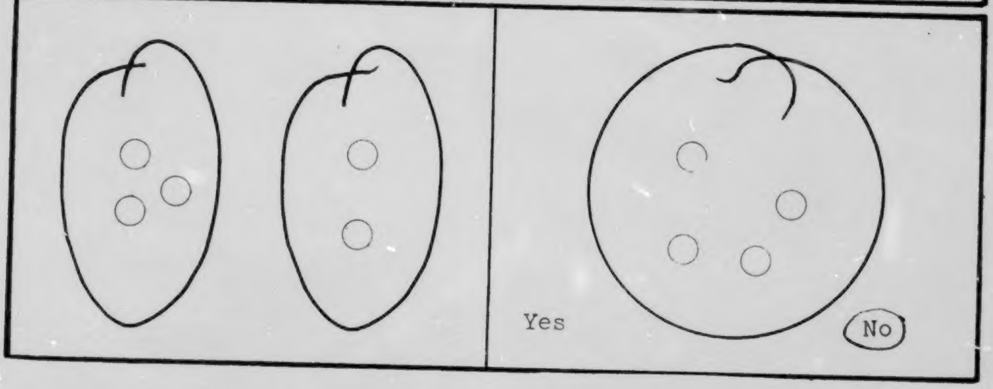

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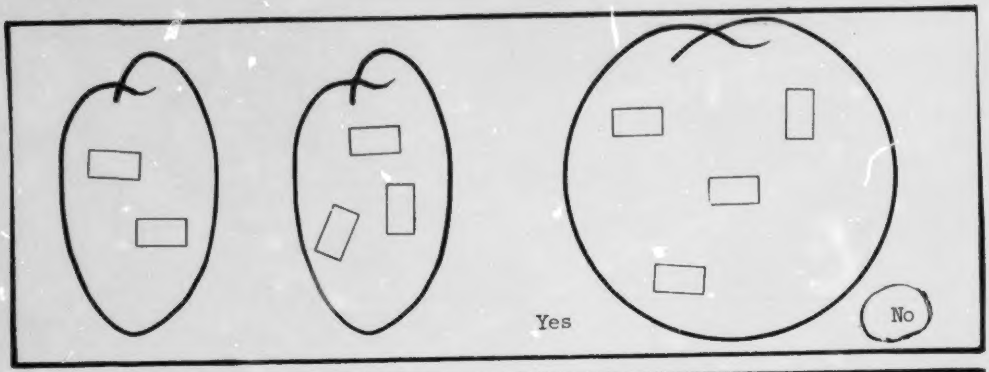
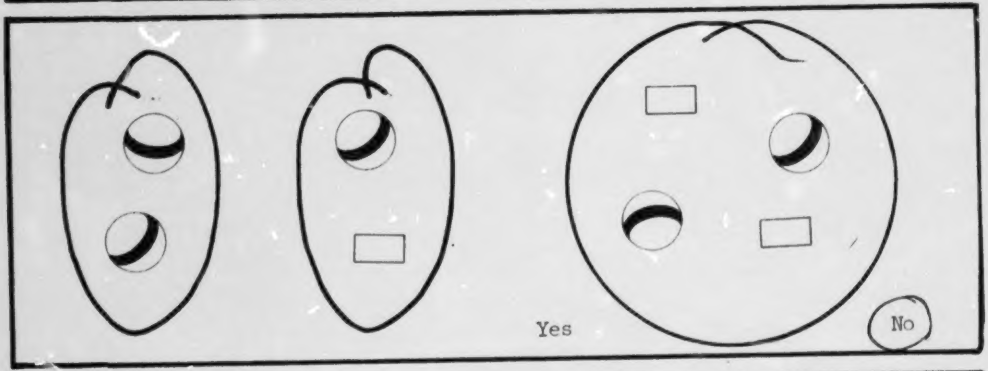


Yes [✓ No]



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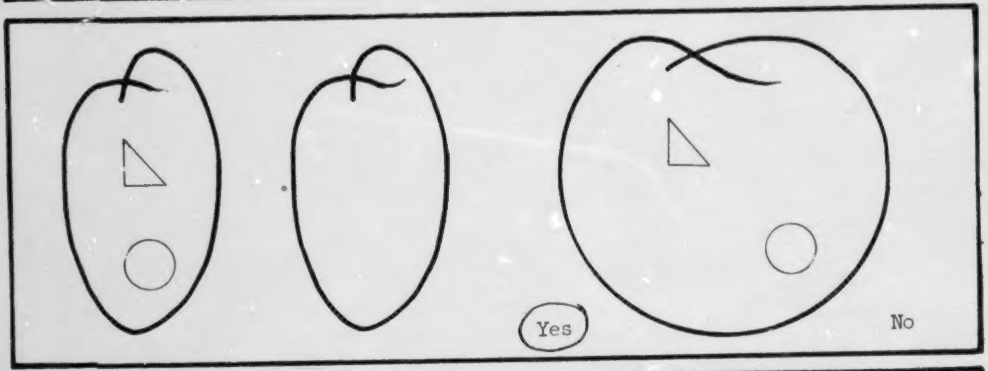

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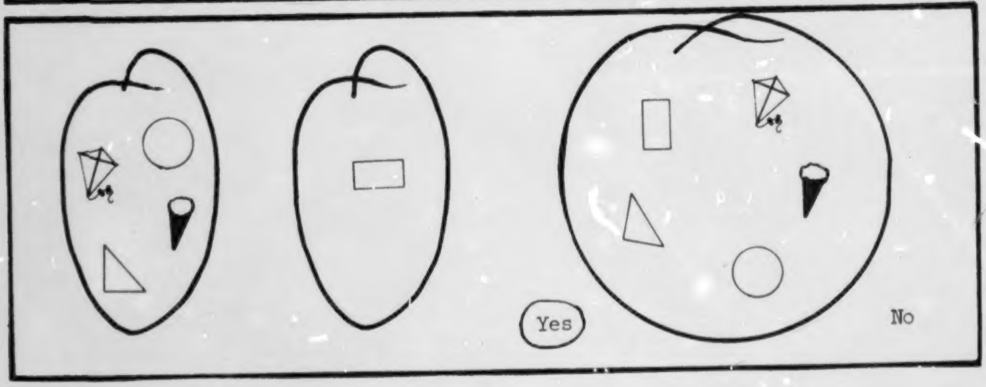


[✓ Yes] No

IV-2. Removing sets

Objective: To introduce the removing of sets in association with the oral use of numbers

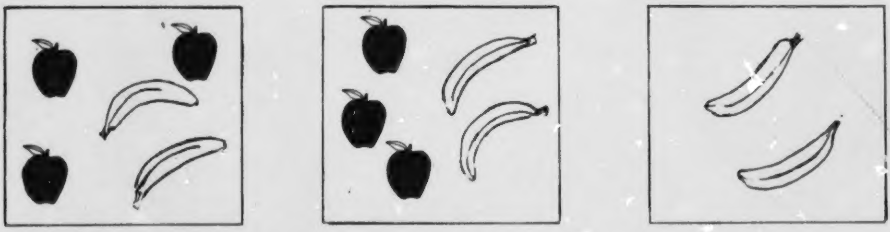
Vocabulary: Review set, member, number, subset, remove, remainder set

Materials: Flannel board materials, bead board or spool board, beads on string, cans of beans, beads, etc.

Suggested Procedure:

The teaching of removing sets by manipulation and using the oral number names can be an outgrowth of the joining process. In the pre-addition exercises the children were asked to identify the numbers of the original sets and the resulting union of these sets. In this pre-subtraction activity, the children first identify the number of original set, then of the subset that is removed, and finally the remainder set. This process will create a natural growth toward the addition and subtraction facts to be presented shortly. Again, the flannel board approach is an excellent source of illustration and introduction.

Place a set of 5 members on the flannel board. Keep these to two readily identified subsets at this time; for example, 3 apples and 2 bananas. Ask the class to identify this set in as many ways as possible: it consists of 3 apples and 2 bananas; has 5 members, etc.. Stimulate the class to take part in the identification and description of sets. The class response can lead to the separation of the set into subsets. At this time count the members of the original set (5). Specify one of the subsets and identify its number (3). By means of manipulation remove the subset and identify the number of the remainder set (2). The illustration below shows the sequence of events previously explained.



The same process may be used varying the sets. The use of the hands in moving the subsets away from the original set helps to establish the idea of removing a set from a given set.

Pupil Pages: Instructions are to be given orally by the teacher with at least one example of the procedure to be used in each activity. The overall objective of these pupil pages is to emphasize identifying the number of the original set, the set removed, and the remainder set.

Pupil Page 20 : There are 6 rectangles on this page. Instructions are to be given with the first 4, the last 2 may be used by the children to create their own set problems. The following guide may be helpful for directing the class:

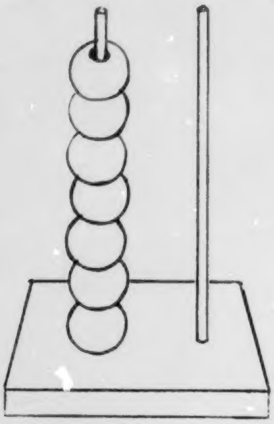
> There are several rectangles on our work page. How many are there? (6) Yes, there are 6 rectangles but not all of them contain pictures. How many contain sets of pictures? (4) How many do not contain pictures? (2) Now, let's look at the first rectangle. How many members are there in the first rectangle? (3) Draw a ring around a subset of 2 members. Now, place your hand over that subset to pretend that you removed it. How many members are in the remainder set? (1) Yes, there is only 1 member left. (Repeat the activity, naming the number of the original set, subset, and the remainder set that you assign to each rectangle.)

Instruct the children to use the last 2 rectangles to make up some problems like the ones above.

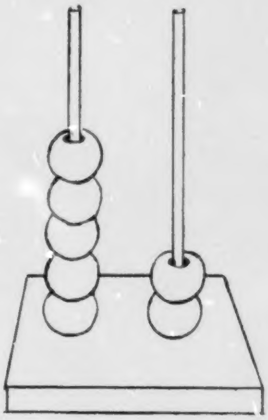
Pupil Pages 21 and 22: Look at the top set on the left. Describe it. Think about removing the subset of triangles. Draw in the upper ring on the right a picture of the subset of triangles you would remove. In the ring below, draw a picture of the remainder set. Do all the activities on these pages in the same way. Look just outside each upper ring to see a picture of an object in the subset you would remove. Note that activity 2 on page 22 includes the removal of the empty set.

Further Activities:

Note the list of activities for Chapter IV, IV-1 and apply these inversely to the pre-subtraction activities. An additional variation includes the construction of a bead board (spool board) with two rods. The board may be used in either addition or subtraction exercises. In the pre-subtraction exercise place a set of five beads or discs on a rod. (Diagram I) Have a child count the original set on the rod and remove a subset of his choice. This subset is counted as it is placed on the rod next to the original set. As the last step, count the remainder set. (Diagram II) This activity may be used by the children during their free time.

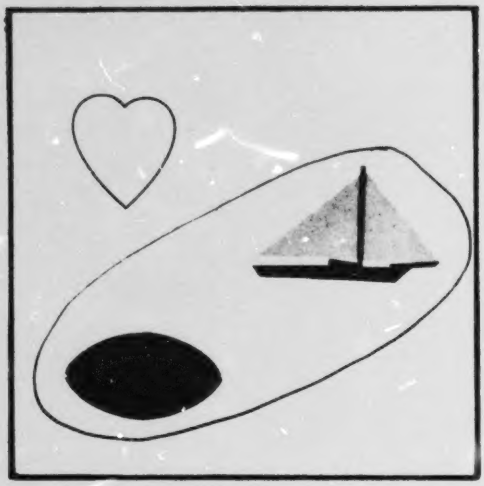


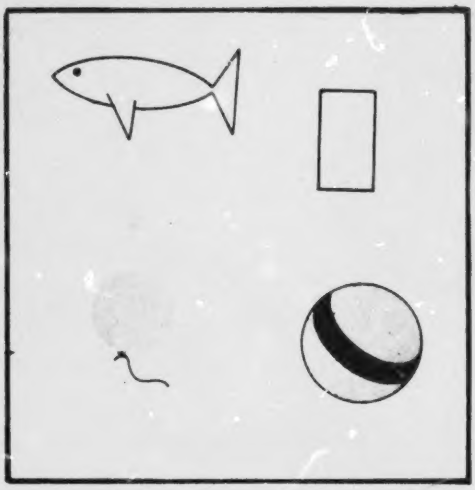
I. Initial position

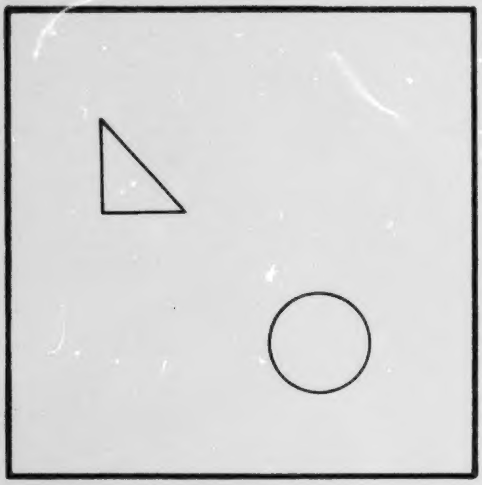


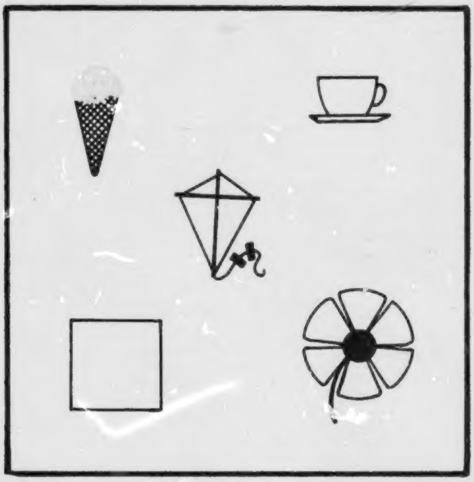
II. Final position

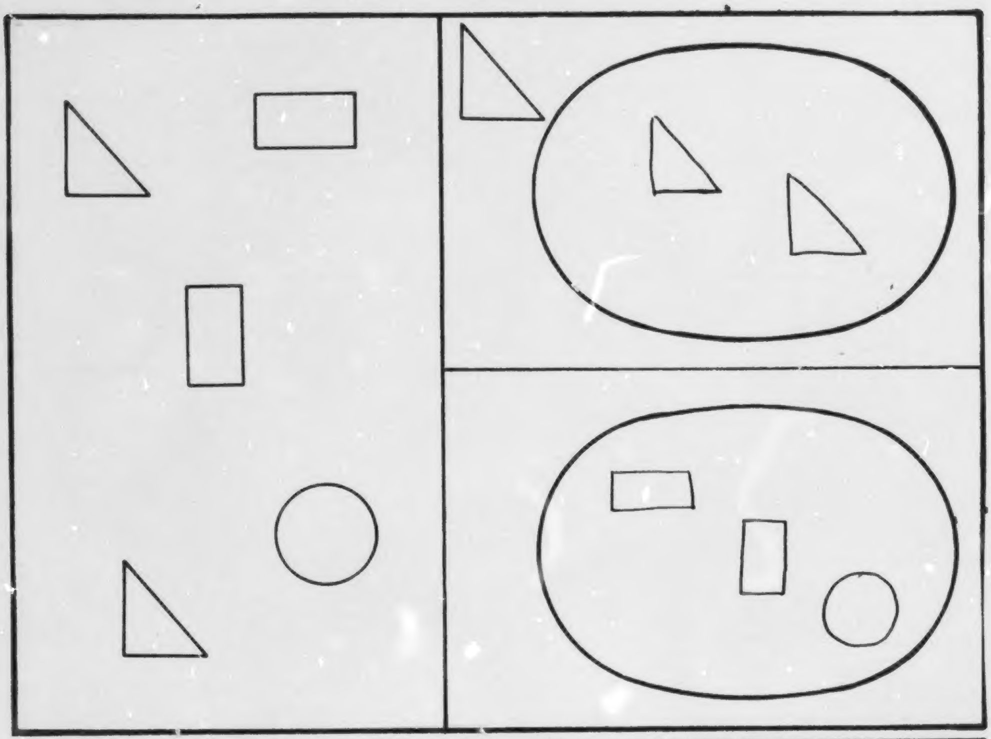
The same method may be applied to beads on a string; use separate jars of beads, beans, etc. with one for the original set and one for the set removed.

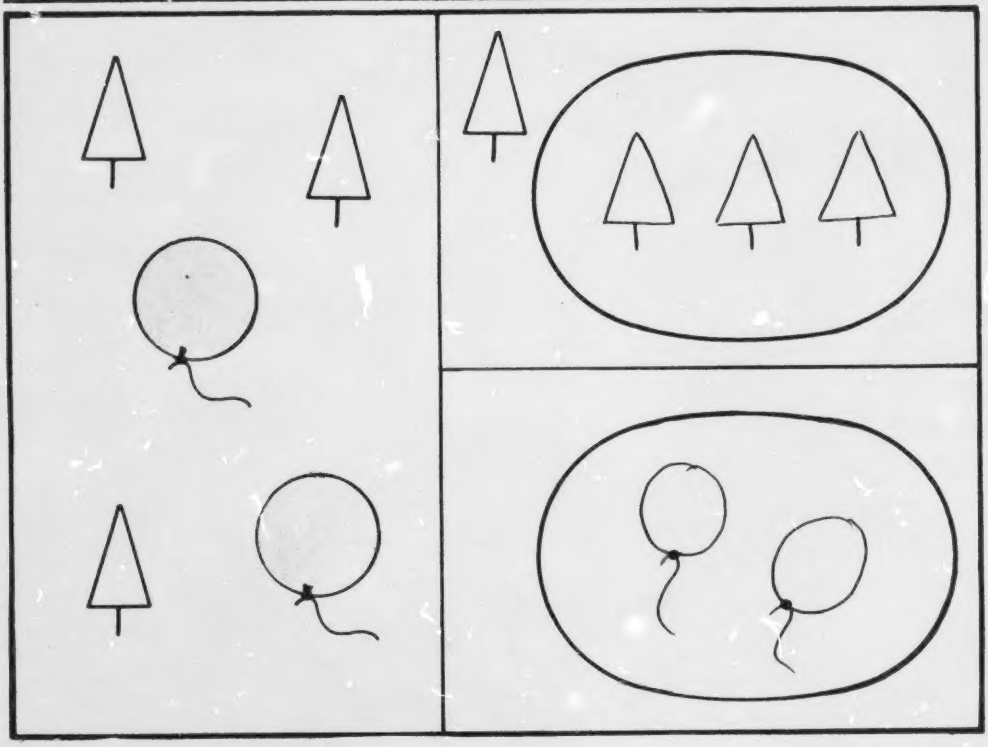


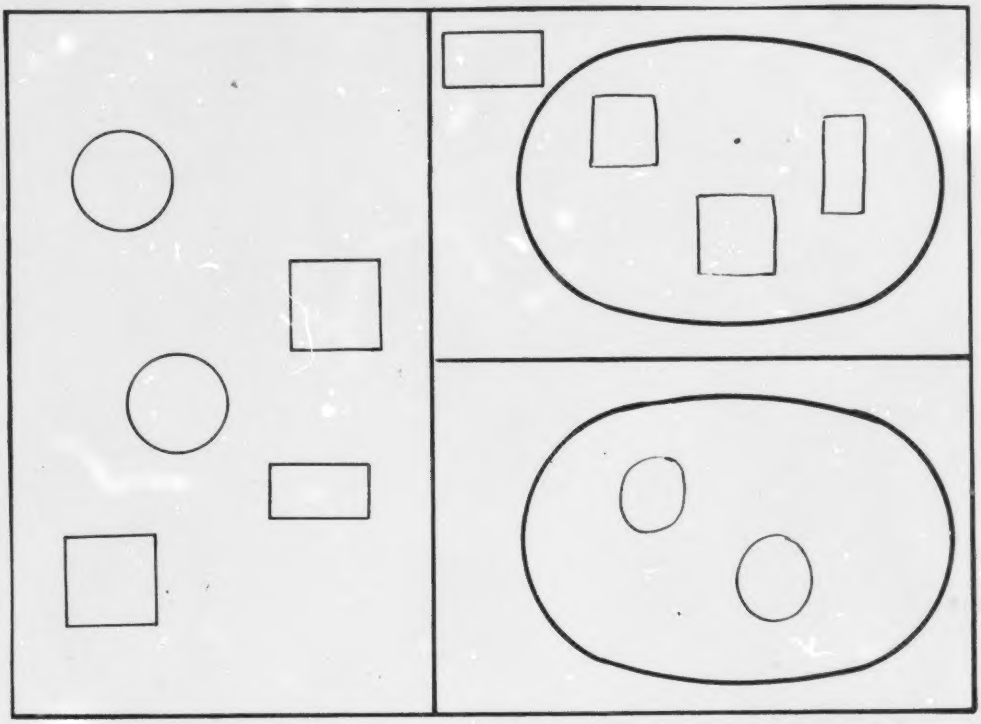


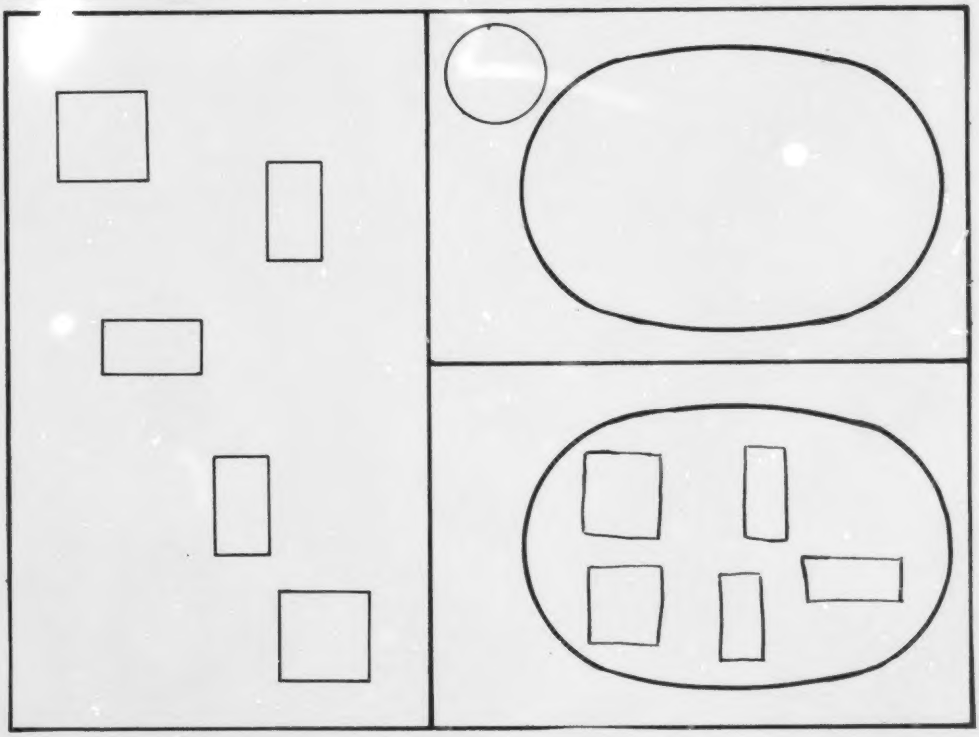












IV-3. Joining and removing a set

Objective: To introduce the joining and removing of a given set as inverse operations

Vocabulary: No new terms

Materials:
1. Sets of physical objects
2. Individual set envelopes or boxes
3. Flannel board and objects

Suggested Procedure:

Both processes involved in this lesson have been previously introduced as separate procedures (joining, removing). At this time the teacher is to demonstrate that the joining and removing of a given set are inverse operations. Since the term "inverse" is difficult at this age level, work at first without using this term. Reinforce the concepts individually and then develop first joining and removing, then removing and joining. In each instance name the number of the original set, the subset to be removed or the set to be joined, and the remainder set or the union. The following example as an introduction again uses the flannel board.

On the left side of the flannel board place 3 objects. Have the class name the numbers of this set and refer to it as the starting set. Place a set of 2 members on the right side and give its number.

> We now have 2 sets on our flannel board. What can we do to find the union of them? (Join one set to the other set.)
>
> Let's join the set with 2 members to the set with 3 members. (Show by moving the one set with the other.) We now have one set of how many members? (5)

Once the union is identified and its number named, have the children remove the subset of 2 and count the remainder set. At this point stress that we again have the starting set. (3) Help the children to understand that joining a set with another set and removing the same set results in the starting set. Continue this type of activity with different combinations remembering that the union cannot exceed 5 members. When joining and removing is understood, proceed to removing and joining. Begin with a set of 4 members and remove a subset of 3. Count the remainder set. Join the subset removed to the remainder set and form the starting set again. Stress that removing the subset from an original set and joining it again to the remainder set results in the original set. Relate number orally to the sets involved. Include joining and removal of empty sets. At the beginning of the flannel board work, it would be best to use sets of like members (shapes, colors, etc.) to help differentiate the sets visually as well as by counting. Combining different objects after this point will help to stimulate thinking and interest.

Further Activities:

1. Place a set of physical objects on the table. Identify these objects and count the number of members. To help define the set when small physical objects are used (shells, rocks, rubber bands, etc.) put them on a sheet of colored paper. Once the original set is identified, put another set in like manner on another piece of paper. Upon proper identification of this set join the one set to the other and count their union. Remove the joined set from the union and count the remainder.
2. The individual set envelopes or boxes should be prepared with set members made either by the children or by the teacher. Use the procedure in developing joining and removing as was used in No. 1. Enabling the children to manipulate their own objects on the desks offers more opportunity for creativity and understanding.

IV-4. Comparing sets

Objective: To extend the children's understanding of "more than" and "fewer than" to the concept of "how many more" and "how many fewer," using oral names for the numbers 0 through 5

Vocabulary: How many more, how many fewer. Review: more than, fewer than

Materials: Flannel board materials, individual set materials, tom-tom, handfuls of colored pegs

Suggested Procedure:

Construct on the flannel board, and have each child construct on his desk a set having 5 members. Construct to the right of the set on the flannel board a set having 3 members, and have each child construct an equivalent set on his desk.

> Let's try to pair off the members of this set (pointing to the set of 5) with the members of this.

On the flannel board, pair 3 of the members of the first set with the members of the second, and ask the children to do the same at their desks.

> Are the two sets equivalent? (No)
>
> *How many more* members has this set (pointing to the set of 5) than this set? (2) *How many fewer* members has this set (pointing to the set of 3) than this? (2)
>
> How many members must I join to this set (pointing to the set of 3) to get a set which is equivalent to this (pointing to the set of 5)? (2)
>
> How many members would I have to remove from this set of 5 members to get a set equivalent to the set of 3? (2)

Select other similar experiences as necessary. In each case start with two sets, and attempt to set up a one-to-one correspondence, noting both the number of members that must be joined to the smaller set to obtain equivalent sets, and the (same) number of members that must be removed from the larger to obtain equivalent sets.

Further Activities:

Stand behind the children and ask them to listen carefully as you beat a tom-tom. After you have finished 2 beats, ask each child to place colored pegs on his desk to make a set equivalent to the set of beats he heard. Now beat the drum 4 times. Have each child make a separate set of 4 pegs on his desk.

> Which of these two sets has more members? (The set of 4)
>
> How many more pegs are there in the set of 4? (2)
>
> How many pegs must be joined to the set of 2 to get a set equivalent to the set of 4? (2)
>
> How many pegs would have to be removed from the set of 4 to get a set equivalent to the set of 2? (2)
>
> Now join the set of 4 to the set of 2. Can anyone tell me how many members there are in the union of these two sets? (6)

IV-5. Partitioning sets

Objective: To teach the partitioning of sets up to 5 members,

Vocabulary: Partition

Material: Flannel board material, colored paper shapes for individual set material, sheets of white 9" × 12" paper

Suggested Procedure:

Develop by means of a flannel board demonstration the meaning of the word "partition": separate a set into two subsets, using yarn or any other convenient device. Observe how many members are in the entire set, and how many are in each of the subsets. Repeat the procedure, partitioning the same set in different ways.

Have each child place a 9" × 12" sheet of paper on his desk and place four circular shapes on the paper.

> Lay your pencil on the paper so that you have some members of the set on one side of the pencil and some on the other.
>
> What could you call the part of the set on each side of the pencil? (A subset)
>
> Fred, how many members are in your whole set? (4)
> How many members are in each of your subsets?
> (Answers will vary.)
>
> Glenn, how many members are in your whole set? (4)
> How many members are in each of your subsets?
> (Answers will vary.)

Continue until all possible partitions of a set of four have been found. (4 and 0, 3 and 1, 2 and 2, 1 and 3, 0 and 4)

During the next few days continue activity of this sort until each set of at most 5 members has been partitioned in every possible way. Identify, for each partition, the number of members in the whole set and in each of the two subsets.

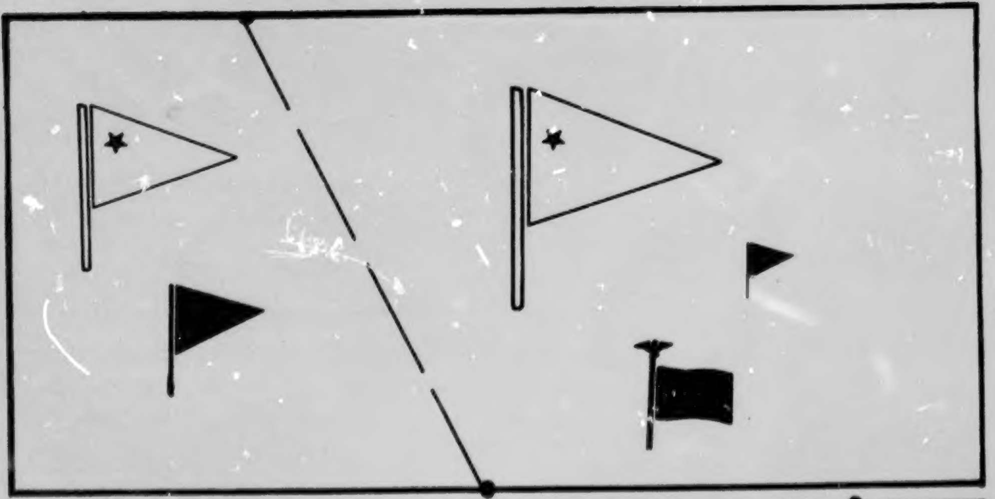
Pupil Page 23: Have the pupils imagine that each rectangle is a "fence" around a set of things. The dots represent "fence posts." Each set is to be partitioned by drawing a line from one fence post to the other, as indicated by the dashed line in the first example. Have children trace over the dashed line in the first example, and then draw straight lines between the fence posts in each of the other two examples. Discuss each partitioning, having pupils indicate the number of members in the whole set and the number of members in each subset.

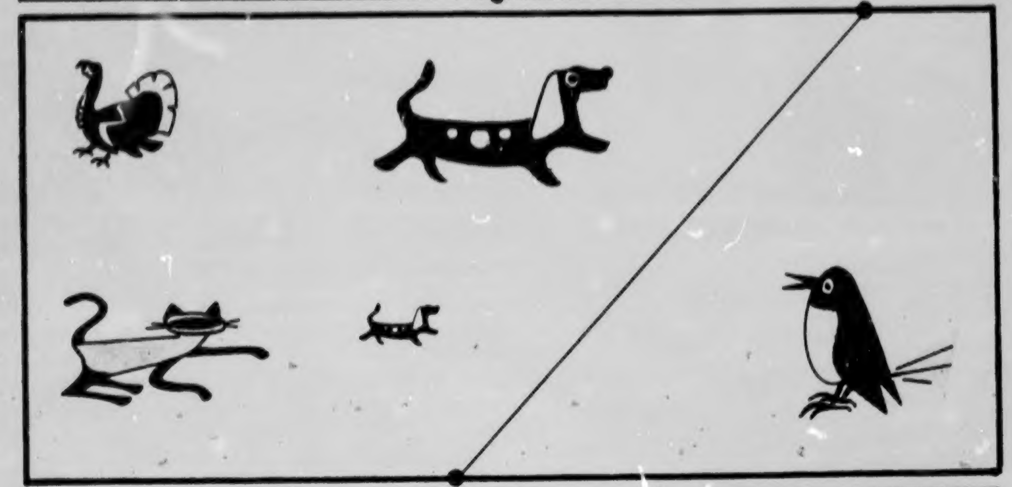
Further Activities:

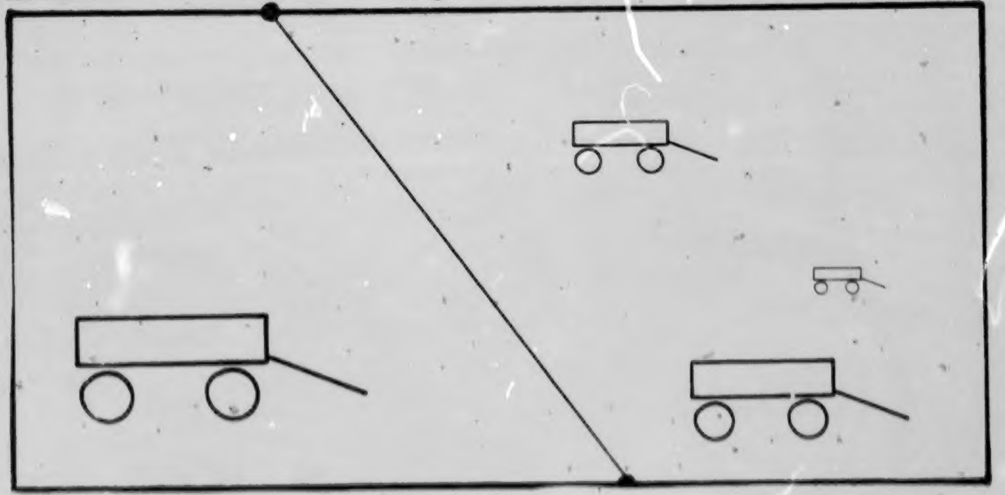
1. Duplicate sheets with geometric shapes (not more than 5) within each "fence," but do not show any fence posts. Have each child mark two fence posts and partition each set as he wishes. Then have children work in pairs, exchanging papers and identifying the partitions made by each other.

2. Suggest that children may wish to use set materials at their desks to help them with this activity.

   Tell the pupils that you are thinking of a set with 5 members--that your set is partitioned into two subsets--and that there are 3 members in one of the subsets. Ask how many members are in the other subset. (If necessary, demonstrate on the flannel board how children could use their set materials to answer your question.) Repeat with other partitions of sets with no more than 5 members.







Chapter V

NUMBERS AND THE NUMBER LINE

Background

The purpose of this chapter is fourfold. First, we wish to teach the pupils to recognize and to write the numerals: 0,1,2,3,4,5,6,7,8,9. Each of these symbols must be connected with the correct spoken word and with those sets having the correct number of members. It is worth noticing that pupils in the first and second grade have to establish the association of four different sorts of objects with each number. For example, the pupil must learn to connect the number 8 with sets having eight members, with the symbol "8," with the written word "eight," and with the spoken word "ate." At the same time the student is learning other names for 8, such as $4 + 4$, $9 - 1$, and so on. Each of these associations must be established by a good many different experiences.

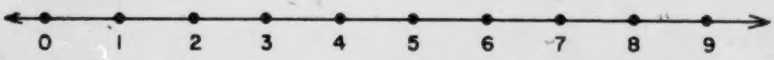
There is one special caution with respect to this chapter. We use the spoken word for ten, and you may want to use the written word "ten," but you should not use the numeral "10." Effective use of the chapter on place value in this text requires that we delay the formal use of this symbol until we reach the subject of place value.

Our second task is to reinforce the concept of arranging sets in order and to use this notion to introduce inequality of numbers. We recall that sets may be arranged in a natural order. One set has fewer members than another if, when the members of the first set are paired off with the members of the second, there are members of the latter left over. (We also stated this notion in another way: one set has fewer members than a second if there is a one-to-one correspondence between the first set and a proper subset of the second.) This ordering of sets shows us how to order numbers.

We say that 5 is greater than 3, and that 3 is less than 5 because a set of 3 can be put into one-to-one correspondence with a proper subset of a set of 5. In other words, if the members of a set of 3 are paired off with the members of a set of 5, there will be members of the latter set left over. Notice that if one set has *more* members than another, then the number of the first set is *greater* than the number of the second set; if the first set has *fewer* members, then the number of the first set is *less* than that of the second; and if sets are *equivalent* then they have the *same* number. The words "more than," "fewer than," and "equivalent" refer to sets. The corresponding notions for number are "greater than," "less than," and "equal."

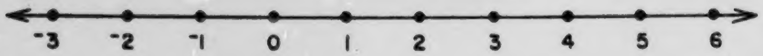
Third, we reinforce the children's concept of inequality by discussing "one more" and "one less," and relating this to joining a one member set to a set and removing a one member set from a set. We attempt to establish a clear picture of sets arranged in order, the sets having 0,1,2,3,4,5,6,7,8,9, and ten members, with all of the numerals firmly attached to the correct set (except for "10"), and with the successor and predecessor of each number clearly visible.

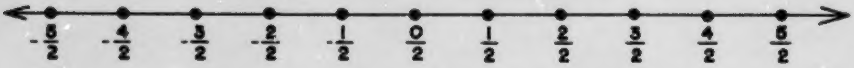
Lastly, this chapter begins the development of the concept of the number line. At this stage we are trying to lead the pupil to think of the numerals 0,1,2,3,4,5,6,7,8 and 9 as being attached to points (or dots) which are equally spaced on a line. We put arrows on the ends of the line segment we draw to try to indicate to the students that there may be other points and labels which might fit if we had a longer line segment. Eventually we want them to conceive of a line which extends indefinitely in both directions, but the picture we want to give now is the following:

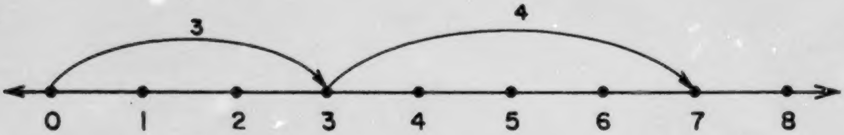


Be sure that they understand that the numbers are associated with the *points* on the line, not with the intervals. Also be sure the labeled points are spaced equally, because, among other things, we are leading up to the notion of a ruler and of measurement.

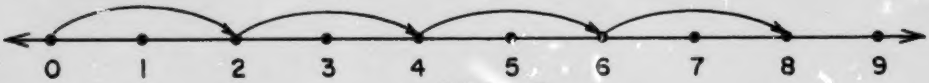
The number line is important, both mathematically and pedagogically. In later grades we will show pictures for the number line which look like these:







3 + 4 = 7



4 × 2 = 8

Without going into details, these pictures indicate that the number line will be used to motivate the introduction of negative numbers, to motivate the introduction of fractions, to emphasize that one number may have many names, to illustrate the notion of addition, and to give an interpretation of multiplication. There are many other pedagogical uses for the concept: indeed, any of the foregoing pictures serves to reinforce understanding of the natural order of the numbers. However, there are also profound mathematical reasons for the early introduction of the number line. We are associating an algebraic object (a number) with a geometric object (a point), and this foreshadows the close connection between algebra and geometry which was discovered by Descartes.

V-1. The numerals 0 through 9

Objective: To associate the written numerals 0 through 9 with their corresponding numbers, and to order these numerals in accord with the natural order of sets

Vocabulary: Symbol, numeral

Materials: Spool board, flannel board, numeral cards (0 - 9), 5 blocks, 5 books, 5 counters, 5 brushes

Suggested Procedure:

Tell the children that many kinds of marks can be used to tell how many. Make a set of marks (/////) and tell a child to bring a set of books with that many members to the front of the room. Ask another child to bring a set of blocks whose number of members is the same as this set of marks (o°o°o). Ask another child to bring ☆ counters. Ask another child to get ▲▲▲▲▲ brushes, etc. Discuss the number in each set as it is brought to you. (5) Indicate that each mark is just another name for the number of the set. Marks are *symbols*. Introduce a numeral card for 5. Explain that the figure is called a *numeral*, that it is the special symbol for the number five, and that it is read "five."

Show a word card with Betty (or the story character from your reading series) on it.

> Is this word really Betty? Does the word have curly yellow hair and a smiling face? (No) It is just the way we write Betty's name.

Have ordered sets from 1 to 9 on the spool board. Leave space for the empty set. Have children locate a set of 5 spools on the board. Hang the numeral below the set of 5. Continue with the introduction of other sets and the numeral cards 1 to 9. Ask where the symbol for the empty set belongs. Place it to the left of the numeral 1 on the spool board. The flannel board and chalkboard can be utilized to vary the procedure for interest. Sometimes order the sets from top to bottom; sometimes order them from left to right. Use pegboard or spool board displays to relate sets and numerals. (More attention is given to recognition of sets 6 to 9 in Chapter X.)

Pupil Page 24: Direct children to draw a line from each set to the numeral for the set.

Pupil Page 25: Direct children to draw a line from the numeral 5 to each set of 5.

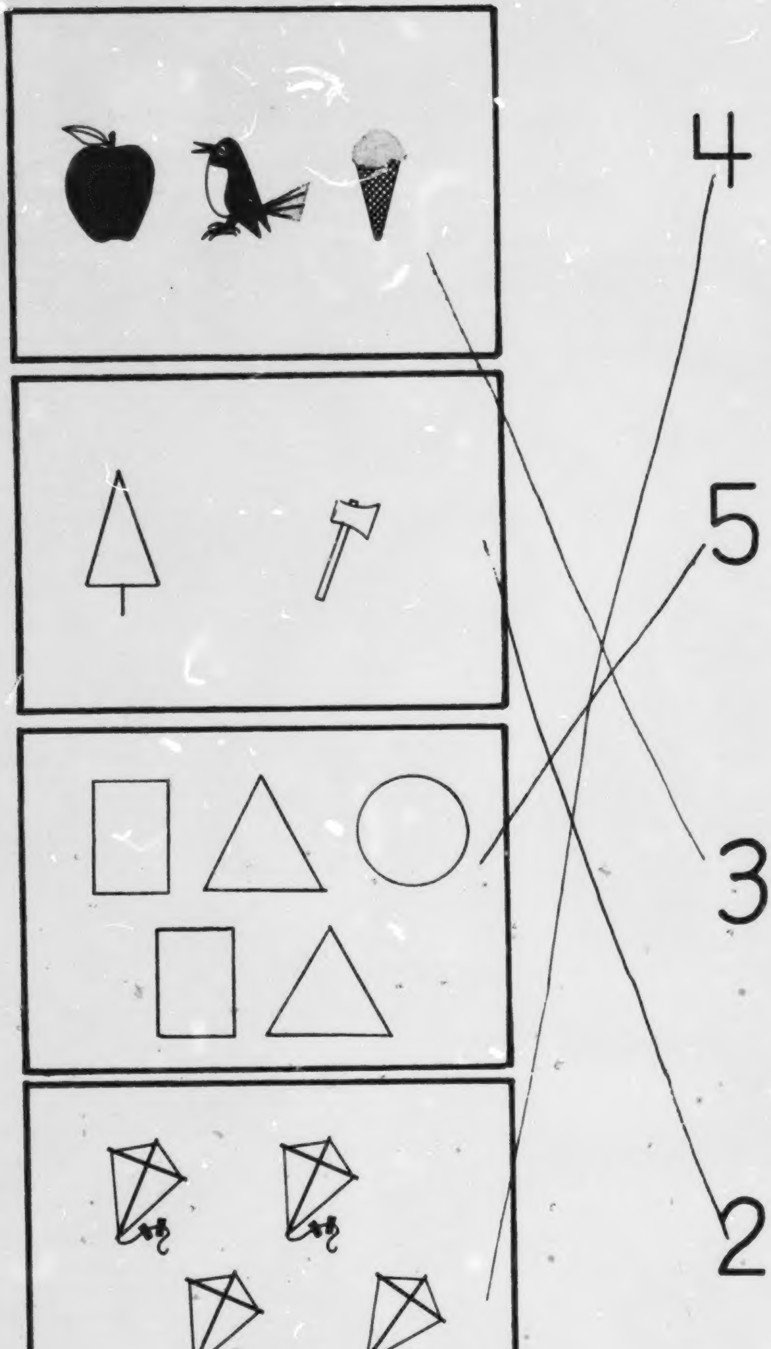
Pupil Page 26: Direct children to draw a line from the set in each ring to the numeral for the set.

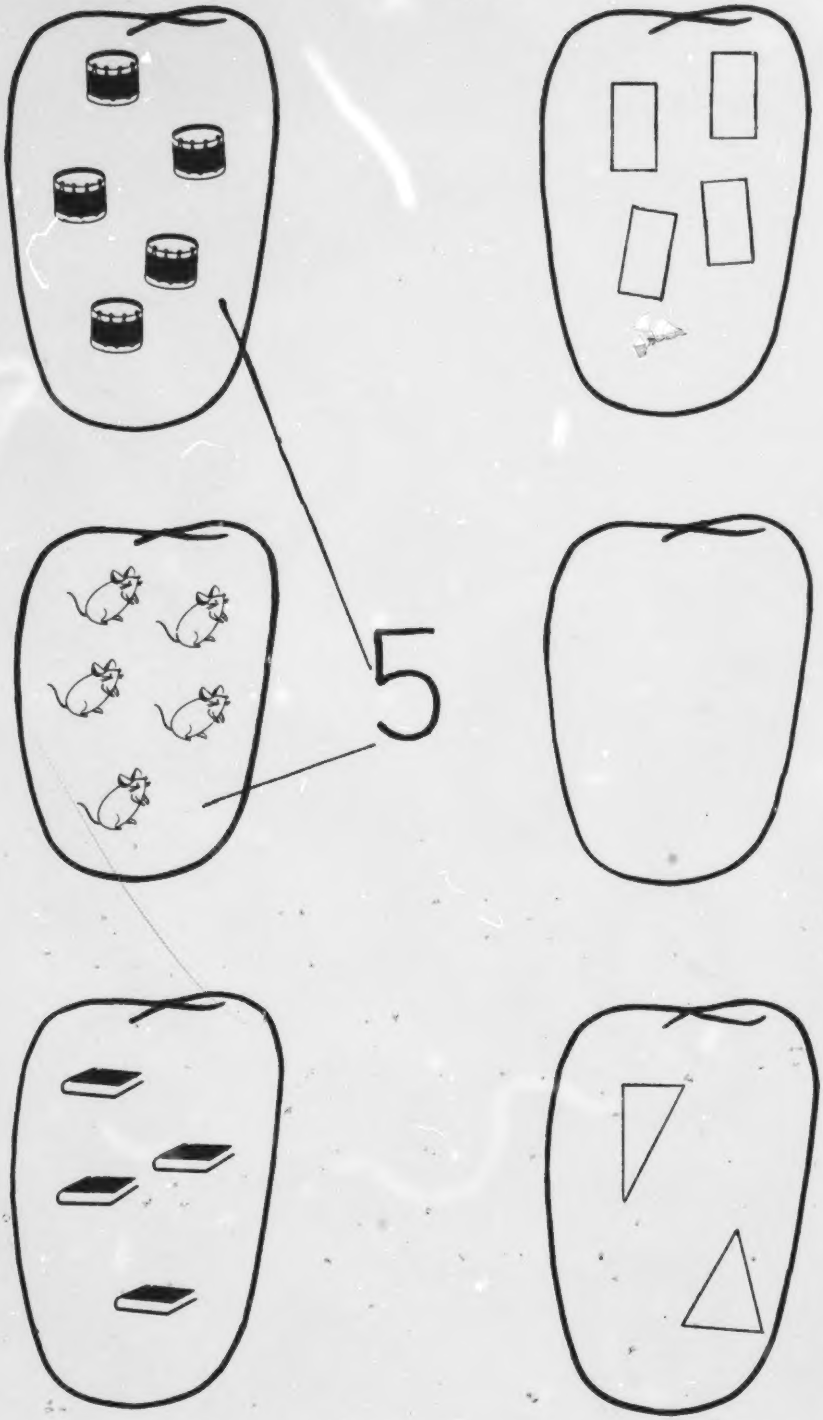
Pupil Page 27: Direct children to count the members of the set and ring the numeral which is the same at the right side of the box.

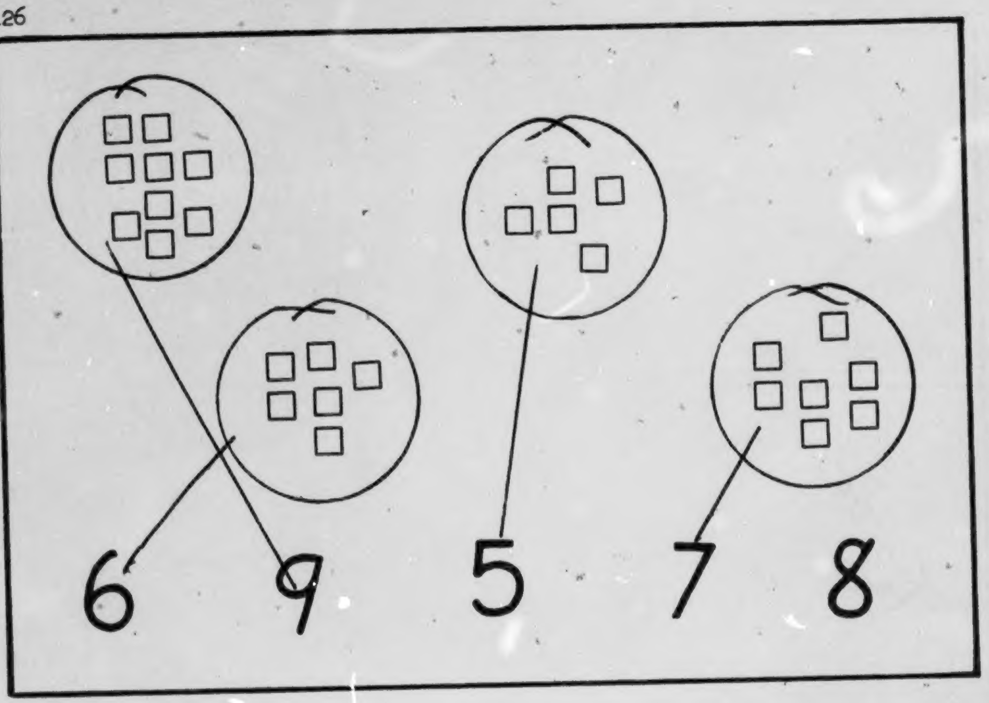
Pupil Pages 28 and 29: Direct children to draw a ring around the numeral for the set.

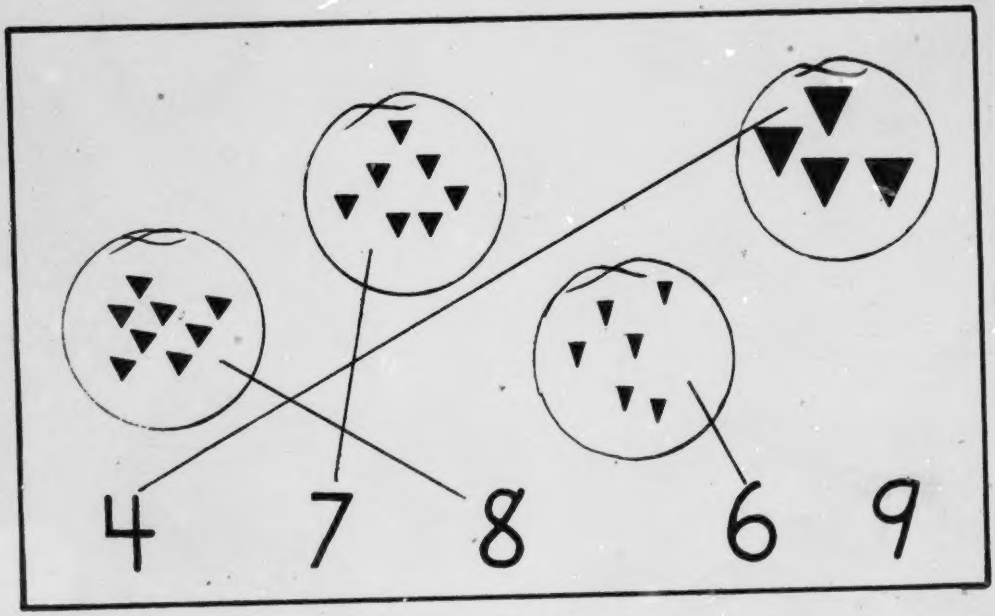
Further Activities:

The writing of the numerals can be taught independently of and following the ability to recognize them. It is recommended that the writing of numerals be taught outside of the time for mathematics in order that the complicated procedure of writing not get in the way of the concept of number itself.


4
5
3
2


5


6
9
5
7
8


4
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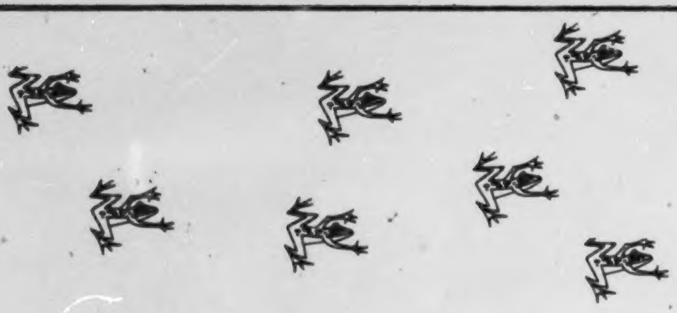
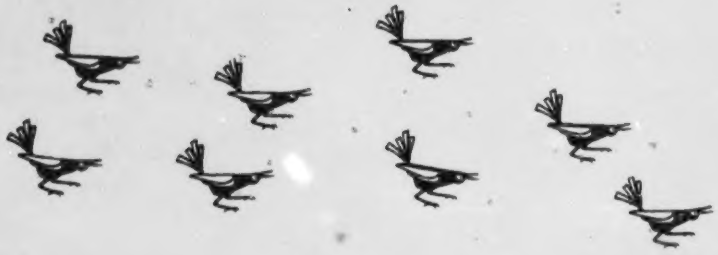
9



2

[✓ 7]

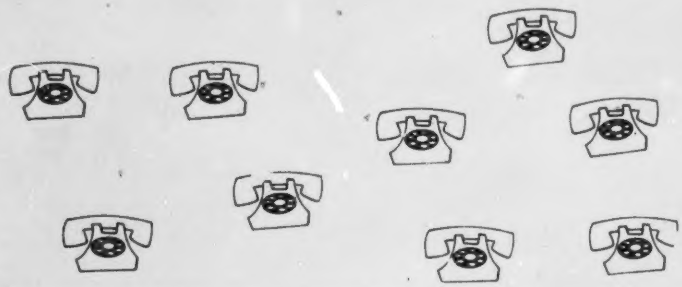
4



[✓ 8]

6

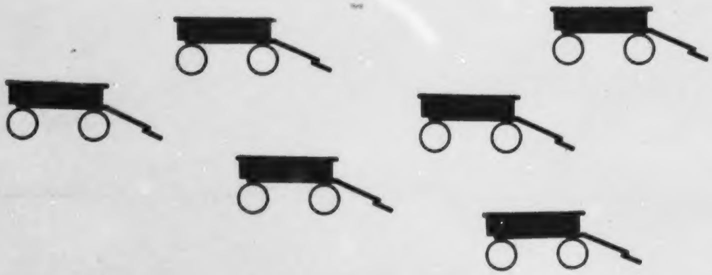
4



3

[✓ 9]

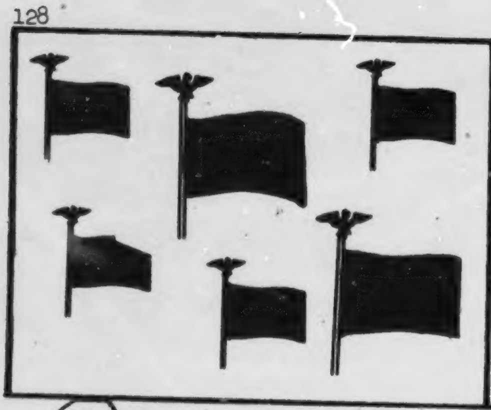
0



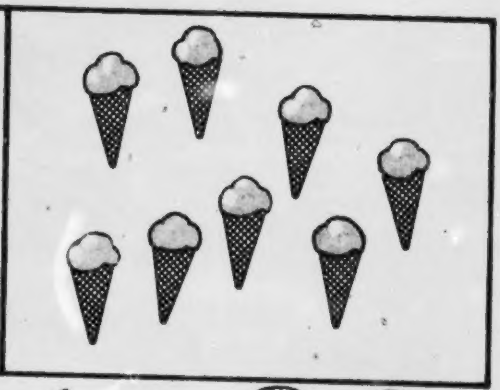
5

[✓ 6]

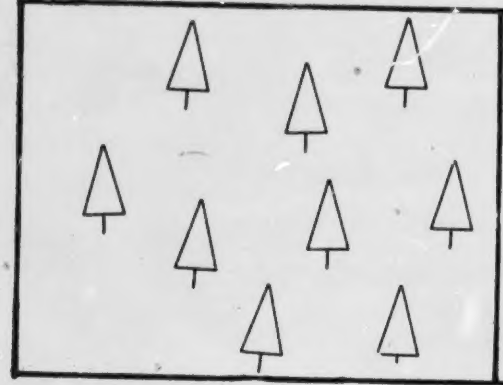
1



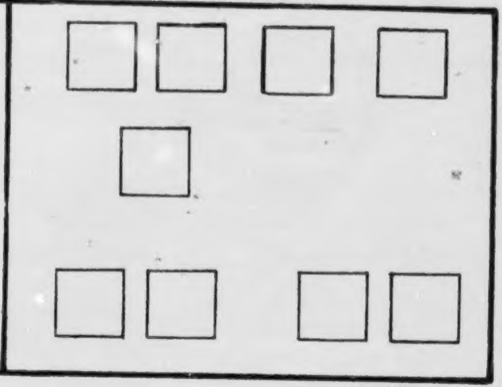
[✓ 6] 7 8 9



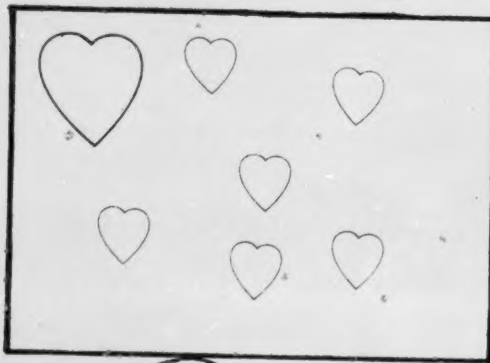
6 7 [✓ 8] 9



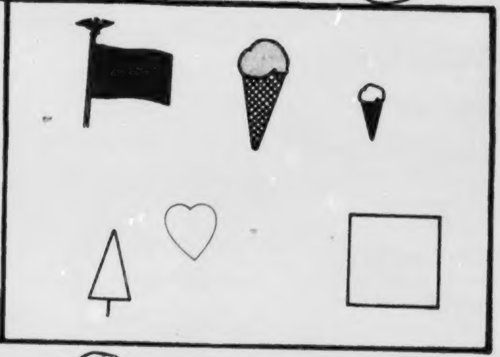
6 7 8 [✓ 9]



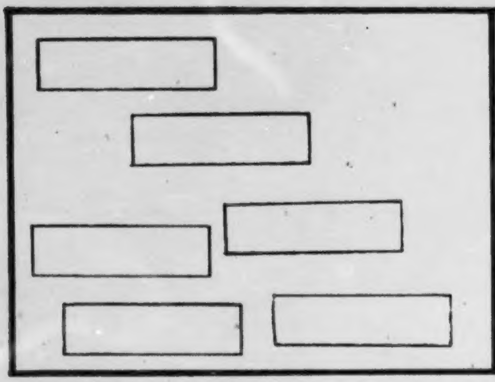
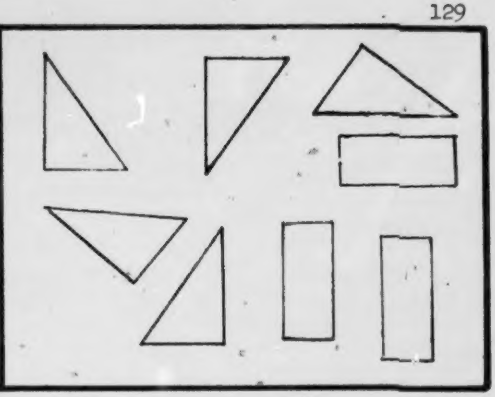
6 7 8 [✓ 9]



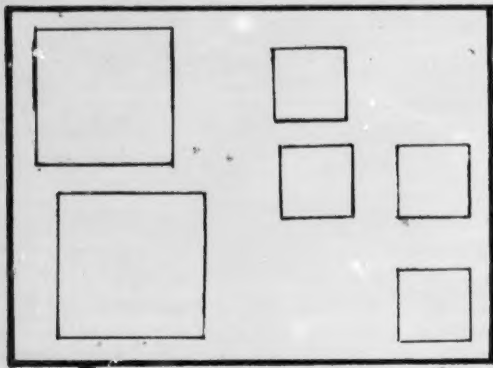
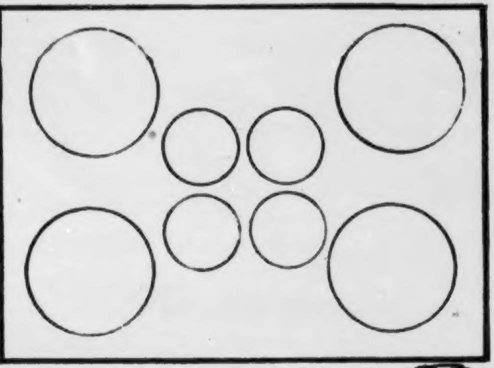
6 [✓ 7] 8 9



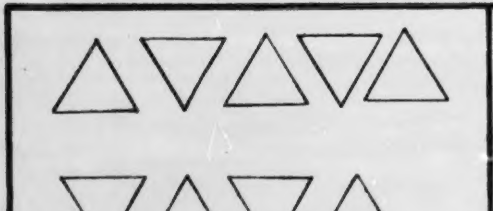
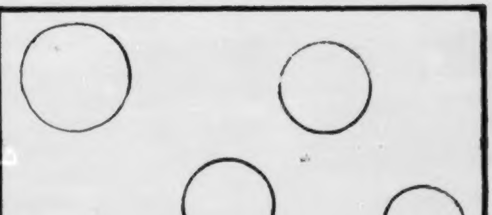
[✓ 6] 7 8 9

8 [✓ 6] 7 9 9 7 [✓ 8] 6

[✓ 6] 9 7 8 7 6 9 [✓ 8]

6 7 8 [✓ 9] 9 6 8 [✓ 7]

V-2. Numbers can be ordered

Objective: To introduce the ideas of greater than and less than

Vocabulary: Greater than, less than

Materials:
Set materials for teacher and children
Set of flannel objects for sets (1 to 9 members)
Numeral cards for children

Suggested Procedure:

Have children build sets in order with 1 to 9 members. Place them at the top of their desks. Indicate that the class is to think about the numbers and their numerals for the sets. Ask the children to put the sets with 2 and 3 members at the front of their desks and then place the numeral cards properly.

> Which set has more members? (The set of 3 members)
> Which number do we think of for that set? (3)
> Which numeral card is used with that set? (3)
>
> Since a set of 3 things has more members than a set of 2 things, we say that the number 3 is greater than the number 2.

Next, show that the set of 2 has fewer members than the set of 3.

> When a set has fewer members than another, we say the number for that set is less than the other number. Is 2 less than 3 ? (Yes)
>
> Which numeral card shows the number that is less? (2)

Put the cards and sets back in order.

> Who can find the numeral card for the number that is one greater than 4 ? one less than 3 ? one greater than 2 ? one less than 1, etc.

Conclude the lesson by asking the children to close their eyes. Then ask them to think of numbers greater than 2. Do not limit them to the number 9 but insist each child explain why his number is greater. (It is a number of a set with more members than 2.

Ask for numbers less than 9.

Look for items in the classroom which are sets of 1 to 9 members.

Pupil Page 30: Have children draw a ring around the numerals for numbers greater than the numbers named at the left of the box.

Pupil Page 31: Have children draw a ring around the numerals for numbers in the row which are less than the numbers named at the left of the box.

Pupil Page 32: Have children draw a ring around the numeral for the greater of the two numbers named in each box.

Pupil Page 33: Have children mark with a check all numerals for numbers in each ring which are less than the number names by the numeral in the center of each ring.

Further Activities:

1. Use displays of sets and numerals around the room.
2. Develop the greater than and less than relationships for numbers 6 - 9.

3	2	[✓ 5]	[✓ 4]
4	0	[✓ 5]	1
1	[✓ 2]	[✓ 4]	0
5	4	0	1
0	[✓ 3]	[✓ 5]	[✓ 2]
2	0	[✓ 4]	1

3	[✓ 2]	3	[✓ 1]	4
	[✓ 0]	5	[✓ 2]	[✓ 0]

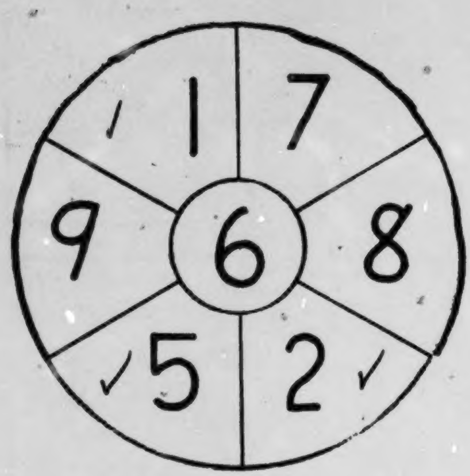
5	[✓ 4]	[✓ 2]	[✓ 3]	[✓ 1]
	5	[✓ 0]	[✓ 2]	[✓ 4]

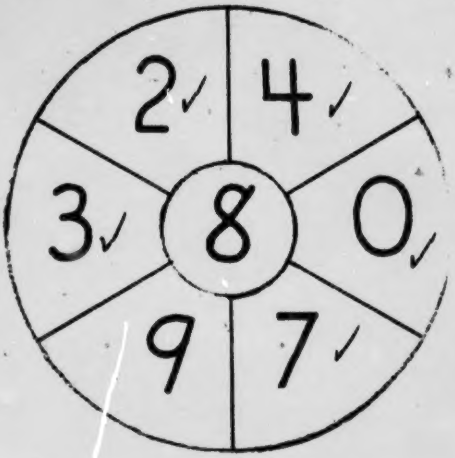
2	5	[✓ 1]	[✓ 0]	2
	3	[✓ 1]	5	4

4	4	[✓ 2]	[✓ 1]	[✓ 0]
	[✓ 3]	5	[✓ 2]	[✓ 1]

1	2	[✓ 0]	4	3
	5	1	3	[✓ 0]

2 / [✓ 4]	[✓ 3] / 1	5 / [✓ 9]
[✓ 2] / 0	[✓ 7] / 6	2 / [✓ 3]
0 / [✓ 9]	5 / [✓ 7]	[✓ 9] / 2
[✓ 8] / 3	[✓ 9] / 8	3 / [✓ 5]
[✓ 4] / 3	[✓ 7] / 5	1 / [✓ 9]
[✓ 6] / 2	[✓ 8] / 0	4 / [✓ 6]


✓ 1
7
9
6
8
✓ 5
2 ✓


2 ✓
4 ✓
3 ✓
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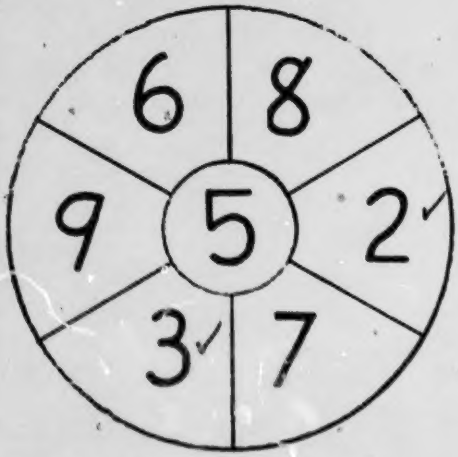
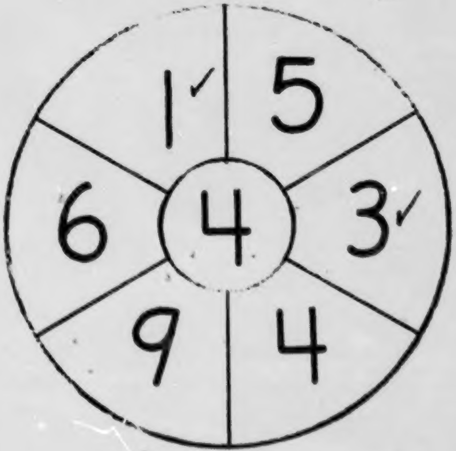
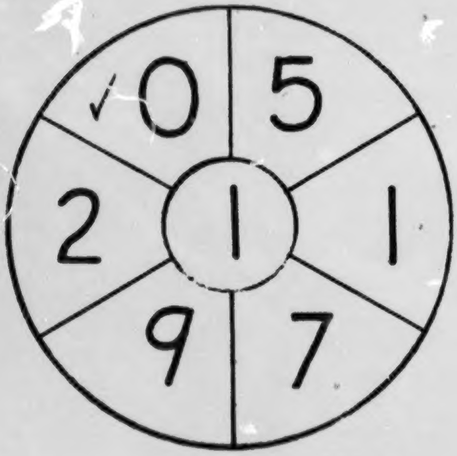
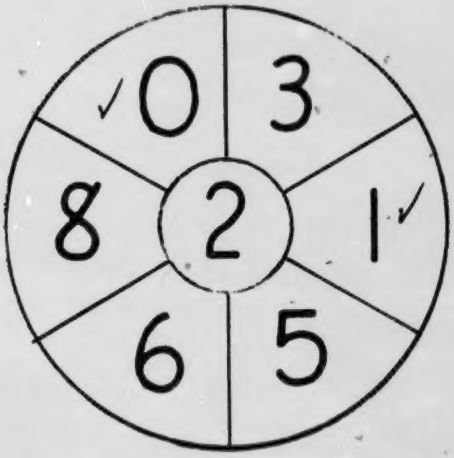
0 ✓
9
7 ✓


2 ✓
3 ✓


1 ✓
3 ✓


✓ 0

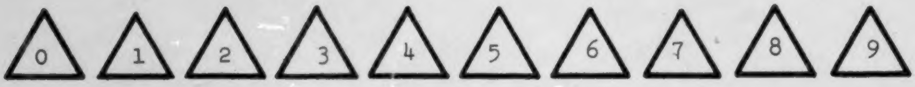

✓ 0
1 ✓

V-3. Linear order

Objective: To develop the idea of the number line

Vocabulary: Point

Materials: Two sets of numeral cards shaped as shown:



Suggested Procedure:

Place numeral cards of one set in random order on the chalk ledge. Ask the shortest child to come to the front of the room. Tell children, "Today we are going to take some steps and count these steps."

> Susie, you stand at our starting point.
>
> (To Susie) How many steps have you taken from the starting point? (None)
>
> What number tells how many steps you have taken? (0)

Place the zero card by the child's feet with the point toward her feet. Tell her to take a step and stop.

> How many steps have you taken now? (1)

Have a child choose the numeral card and place it at Susie's feet. Continue with the same procedure until 9 steps have been taken.

> Would we have to stop with 9 if we had more cards? (No. Could go on and on.)

Call the tallest child to stand beside the first.

> Do you think Bill usually takes the same size steps as Susie? (No) (Warning: Be sure Bill gets the idea that he isn't expected to take the same size steps!)

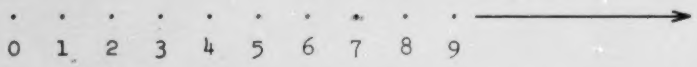
Send Susie to her seat leaving numeral cards on floor and have Bill stand close to, but not on, the place where Susie started. Repeat procedure of placing numeral cards from 0 to 9, using second set of cards. This time use the words, "At the point where Bill stopped," but do not emphasize the word point. Use it casually.

> Which person went farther with 9 steps? (Bill)
>
> Which person took longer steps? (Bill)

Point out the fact that although Bill's steps were longer than Susie's, each of Bill's steps was about the same length as his other steps and that Susie's steps--while a different length from Bill's-- were about all the same length.

Remove the numeral cards from the floor and tell the children to imagine that steps had been taken on the chalkboard. Have a child indicate the starting point and nine steps. Make a dot to show these points. Ask what numeral to write for the point that shows the starting point. (0)

Continue to number points through 5. Have children look at unnumbered points, and ask one to come up and put the pointer on the point where he thinks the next step would be. Ask if points could continue to be made in this direction. Draw an arrow to show more steps which might be taken.



Try to establish for the children, using further experiences if necessary, the picture of equally spaced dots extending indefinitely to the right, with each dot associated with a number.

V-4. The number line

Objective: To continue the development of the number line concept

Vocabulary: Line

Materials: Strong string, set of numeral cards from Section 3, paper clips

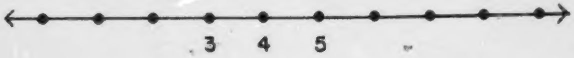
Suggested Procedure:

Have two children, one at each end, pull the piece of string tightly. Ask children to pretend to watch a lady walk across a tight rope. Suggest a spot on the string (to the left of the class) where she begins, and ask what numeral to use at this point. Clip the zero card to the string (using a paper clip at the top of the card).

Have a child come up and stand by the string at that point, take a single step, and "balance" there. Place numerals after children tell you which one for each step.

Ask children to imagine a long, long tight rope with many steps on it. Suggest that it be thought of as a line of numbered steps, all the same distance apart, or a "number line."

Dispense with the string and draw a line on the board. Mark points and label as shown.



Instead of using a tight rope for our number line, let's use a line on the board.

How is this number line different from the one we made on the tight rope? (It doesn't have all the numerals. It doesn't show the first steps.)

Call on a child to use the pointer, show you another point, and tell you what numeral to write there. Have children notice that although not all the numerals are written, they know where others _must_ go because of the ones already written.

Have the other points named and write the numerals (not beyond 9, however.)

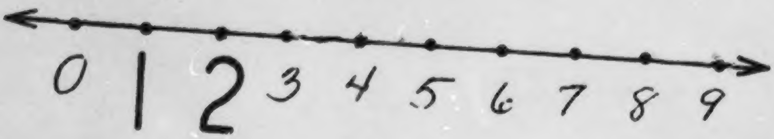
Use the line for children to point to the numeral _before_ 5, the numeral _after_ 1, _before_ 1, etc.

The number before 5 is _less than_ 5, the number after 5 is _greater than_ 5. It should be emphasized that all numbers before 5 are _less than_ 5 and that all numbers after 5 are _greater than_ 5.
Ask children to locate on the number line:

A number greater than 7.
A number greater than 2.
A number less than 4.
A number less than 6.

At a later stage children can be asked to find the number that is two greater than 3; two less than 7, etc.

Pupil Page 34: Call pupils' attention to the fact that the points on one line are all the same distance apart, but the different lines have different distances between points. Ask them to write the numeral for the number that belongs at each point and to place the numeral under the point.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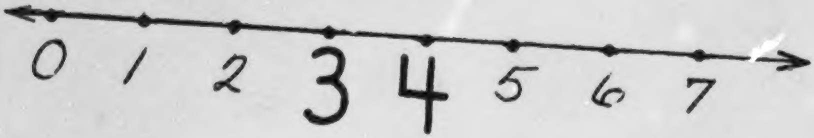
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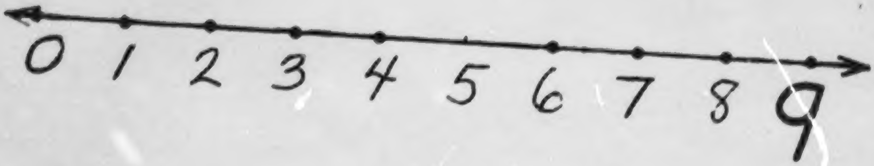
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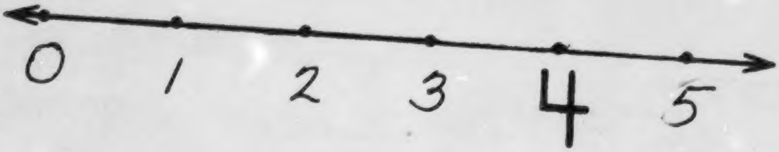
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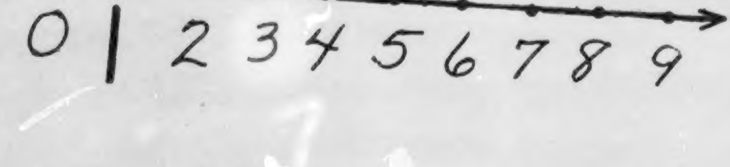
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Chapter VI

COMPARING GEOMETRIC SHAPES

Background

This chapter is an extension of the former one on geometric shapes. Its primary function is to reinforce the pupil's perception of differences and likenesses in shapes and to refine his discrimination among them.

In the former chapter the child has already learned to distinguish and name the triangle, rectangle, and circle. For example, he began to perceive a rectangle as a four sided figure whose corners all "looked alike." He is now asked to identify among the rectangles those whose sides have the same length. These are called squares. This involves the perception of longer than, shorter than, same length as, applied to the sides of a figure. Almost certainly the child will be familiar with both the shape and the name square, but his recognition of just what it is that makes a figure a square and the understanding that a square is a special kind of rectangle are much less likely. There will be many opportunities to identify squares in an outside the classroom environment.

As was emphasized earlier, we wish the words circle, triangle, rectangle (and now square) to indicate curves. That is, they are just "edges." Thus the models that most clearly indicate what is meant are frames of wire or some other material. When any such curve is drawn in a plane there is an intuitively obvious meaning to a mark being inside or outside or on the curve and further emphasis on these terms will aid in emphasizing what the words rectangle, square, etc. actually describe.

As the child looks for examples of the square (and of the other curves) in his environment, many of the examples, such as a base of a block or a cupboard door, actually show not just a square but a square and its interior. As was noted earlier the word region is used to indicate such a figure. Thus a region bounded by a square is a square region, one bounded by a circle is a circular region, etc. The thought is that by providing a word to describe this situation the pupil will be aided in distinguishing the two ideas and will see squares as the boundaries of square regions. The word region was introduced in Chapter II but no particular effort was made to have the children use it. Perhaps now they can use it themselves.

In the latter part of this chapter, the children will have an opportunity to fit together paper or flannel models of various regions. The choice here was to use regions rather than the bounding curves, since these are the easy models to make. Note, however, that if two models of rectangular regions just fit one on the other then their edges (the rectangles) also just fit. Thus it really does not matter whether we say we are fitting rectangles or rectangular regions.

By trying to fit a model of one region onto another, we hope the pupil will be led to make visual comparisons and decide that it is pointless to try to match this figure with that in a certain way because, for example, two segments of clearly different lengths would then have to match or because two corners which would have to go together clearly won't fit. The concept being developed here of two figures being "just alike" is actually that of congruence though the word is not introduced in this grade. Specifically when a model of one geometric figure can be fitted exactly on another they are called congruent. Models of congruent figures are thus interchangeable as far as fitting in a given position is concerned. This is an important idea since it is the basis of much of measurement. For example, to find the length of a segment means to find how many congruent copies of a unit segment are needed to cover it. Similarly, finding the area of a region means counting the number of congruent copies of some unit region necessary to cover it.

The activities here are envisioned as a series introducing whatever degree of complexity the class is prepared to accept. As noted in the suggested procedure, it can start at a level where even the least adept pupil can participate successfully and can go to a stage of geometric perception that would be non-trivial for many adults. Discussion as to whether or not two figures will fit before the fitting is actually tried will be an important part of these experiences.

Care should be taken in these activities that figures be shown in different positions. For example, rectangles and triangles should not always have an edge parallel to the floor. Since so often the examples which the pupil sees in his experience are in these conventional positions, it is necessary to make a deliberate effort to counteract the impression.

The fact that this material can be made quite sophisticated should not mislead the teacher into overdoing it. This is an introduction to an idea which the child will see again and it need not be squeezed dry at the first exposure. It should not be pursued to the point either of frustration or boredom.

In the interests of simplifying the language for these manipulations the teacher may wish to renew with the pupils the earlier "agreement" that the word rectangle will be used for one of the pieces of flannel even though it is really a rectangular region. However, this should not be allowed to obscure the real distinction between curve and region.

VI-1. Comparing lengths of edges

Objective: To introduce the ideas of longer than, shorter than, same length as

Vocabulary: Compare. longer than, shorter than, same length as

Materials: Two brushes (one long or easel brush and one short or paint brush) for each child, three strips of cloth of different lengths, and as needed, yarn, string, pipe cleaners, straws

Suggested Procedure:

Give two brushes to each child, one short paint brush and the other a long easel brush. There should be a distinct difference in length between the brushes. If brushes are not available in quantity, use straws.

Ask the children to put the brushes on end on their desk tops.

> How are these brushes alike? (Both are brushes, wood, etc.)
>
> How are these brushes different? (This brush is shorter than the easel brush; this brush is longer than the paint brush.)

Suggest to a child that he observe the brushes of the child next to him. Ask him to find a brush the same length as one of his, and to display the two. Continue with another child finding a brush longer than (shorter than) his.

Select the children at one table center for demonstration. Distribute a third brush to one child. Ask him to compare the brush with the two members of his set. Seek the response that the new brush is the same length as his easel brush, and longer than his paint brush. Continue by distributing a short brush at the next table center, and so on until each table group has participated or interest lags.

Have children quickly place all brushes in several convenient containers and direct their attention to the flannel board where three strips of colored cloth of distinctly different lengths are displayed. These strips should be placed horizontally with a common beginning position.

Discuss how these objects are alike, how different.

Move the cloth strip of middle length to the side. Ask which of the other strips is longer, which is shorter.

Repeat the experience as needed using display items such as pipe cleaners, straws, or other physical objects. Choose narrow objects to prevent children from examining the total figure rather than concentrating on the idea of comparison of edges.

On the chalkboard uncover horizontal drawings of two line segments of different lengths. Compare as previously using the word "drawing" rather than "line segment" with the children. Point out that when drawings such as these are compared, one drawing will be longer than the other, shorter than the other, or the same length as the other.

Then uncover drawings of two more segments placed horizontally. These should match the first segments. Ask which of the two new drawings is shorter, longer.

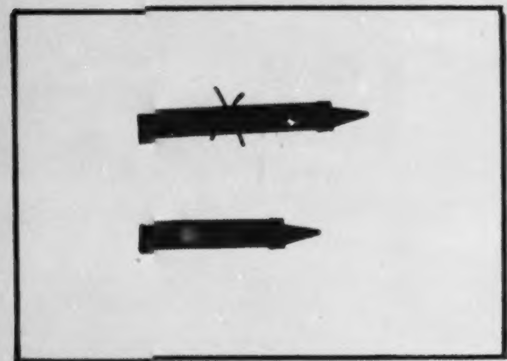
Compare the line segments in both sets. Children will find that one of the segments in the second set has the same length as one in the first set, and that the other also has the same length as another.

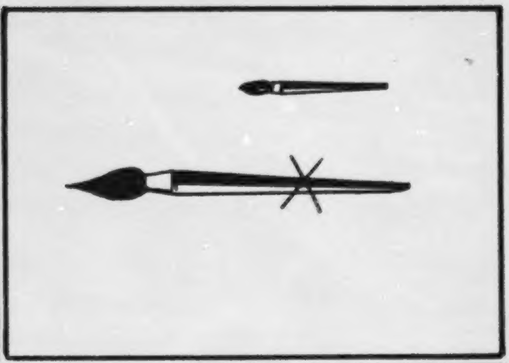
Now uncover a third set of segments that match the first two sets. This set should be located diagonally.

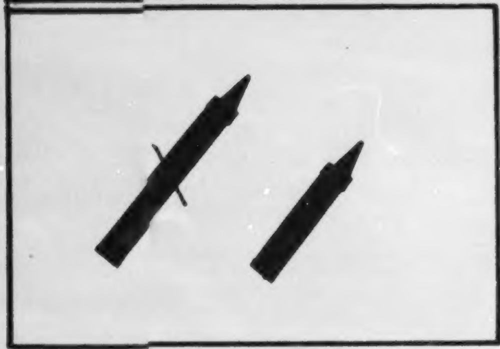
Children may not readily see the comparison, depending on how much attention has been given previously to the varied placement of shapes. Too, it is more difficult to recognize similarities when one set is seemingly out of position. If assistance is needed, use a string or edge of paper to aid in the comparison.

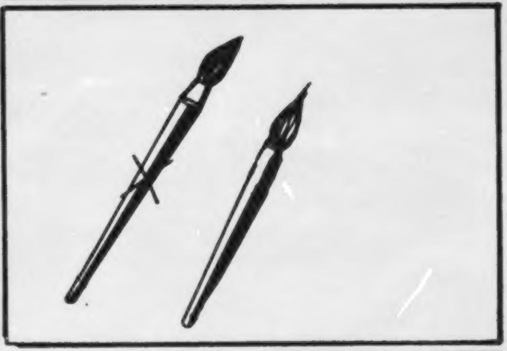
If a string is used, it can also serve the purpose of emphasizing the idea of the "same length as." The ends can be placed together and the string smoothed out. One piece is then the same length as the other.

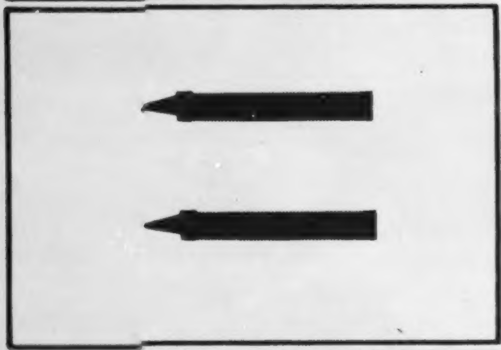
Pupil Page 35: This page is used to evaluate how well children have gained understanding of longer than, shorter than. Instructions should be to mark the member in each set that is longer than another in that set. The bottom four illustrations are more complex, since in one the objects are the same length, in another the direction is reversed, in another there can be two objects marked, and in the last there may be difficulty in determining by sight which is longer. Have string available if the children wish to test the figures.

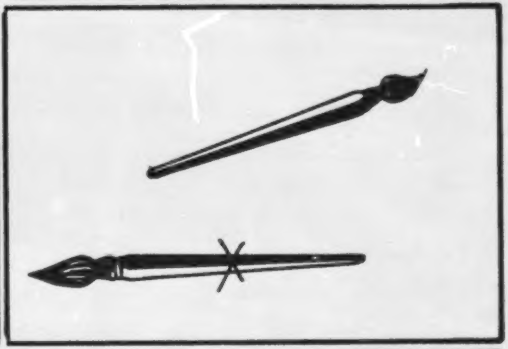


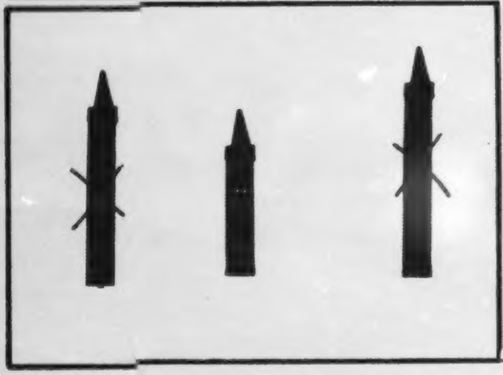


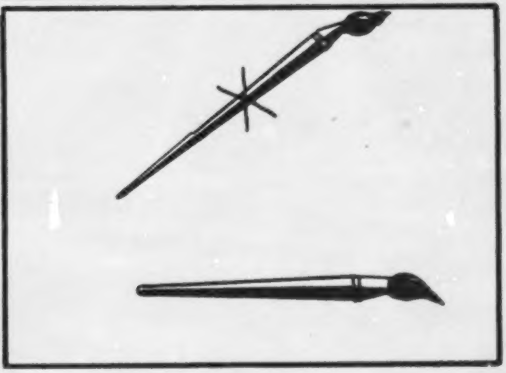












VI-2. Recognizing squares

Objective: To distinguish a square as a special kind of rectangle

Vocabulary: Square, opposite, (review) region

Materials: Wire models of shapes: several rectangles including several squares, parallelogram, rhombus, trapezoid, quadrilaterals; paper or felt models of regions as above

Suggested Procedure:

Display wire models of shapes on a pegboard or bulletin board, and ask children to pick out the rectangles. Recall the earlier description of a rectangle as a four-sided figure that has all four corners alike. Check against this description any shapes erroneously identified as rectangles. If squares are not chosen, have them checked against the description to see that they are, indeed, rectangles.

Remove the shapes that are not rectangles, and leave the squares and other rectangles intermingled on the pegboard. Point to a rectangle and ask whether any two edges appear to be the same length. Explain that these are opposite edges. Show that the other pair of opposite edges are the same length. Have children point out at the board the pairs of opposite edges of all rectangles (including the squares) and note that for all rectangles opposite edges are the same length. Ask whether anyone can find a rectangle that has all four edges the same length. When one square has been picked out, have children inspect it, using a string or piece of paper to compare the length of the sides, if desired, and turn it in several different positions. Explain that rectangles of this sort are called squares.

Have the other squares identified, and separate the squares slightly from the other rectangles. Ask a child to remove those rectangles that have all four sides the same length. Have these replaced, and ask a child to remove those rectangles that are squares.

Display the models of regions on the flannel board or bulletin board (Be sure to place some of these in such a way that no edge is parallel to the floor.) and have rectangular regions pointed out. Remove the models that are not rectangular, have the rectangular regions described, and have the rectangular regions with all four sides the same length removed. Ask what special name might be given to a region whose edges form a square. (Square region.) Have the square regions removed.

Review the distinction between square and square region by observing a wire model of a square (the edges of the region form the square, the region includes both the edges and the inside, etc.)

Note: This may be a good time to stop with some classes or groups of children. As in any of these activities, the teacher must keep the total teaching group in mind and must not assume that the continued enthusiasm of a few means that all are actively interested. Repetition of the ideas should come in many contexts outside the mathematics period. Children can gradually be led to the use of more precise language, etc.

If desired, it may be pointed out that in some figures (parallelogram, rhombus, etc.) opposite sides are the same length, but that these are not rectangles, because they do not have four corners alike.

Before using the Pupil Pages, put all the rectangles back on the display board without other shapes and again have the set described as a set of rectangles of which the set of squares is a subset.

Pupil Page 36: This page is designed to evaluate the understanding of a square as a special kind of rectangle. Instruct children to discover where there are rectangles, each of whose edges are the same length. Ask the children to color the edges of these squares.

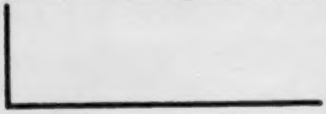
Pupil Page 37: Have children color edges of all rectangles blue. Have them make an × inside each rectangle that is a square.

Further Activities:

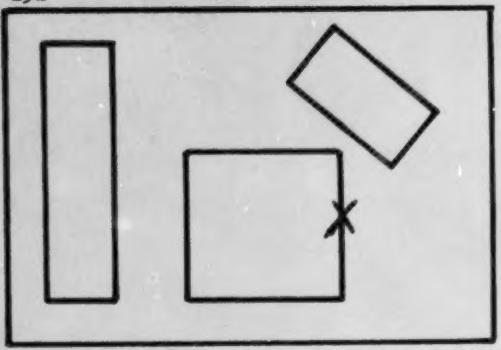
Draw on the chalkboard two congruent line segments that could form one corner of a model of a square, as:

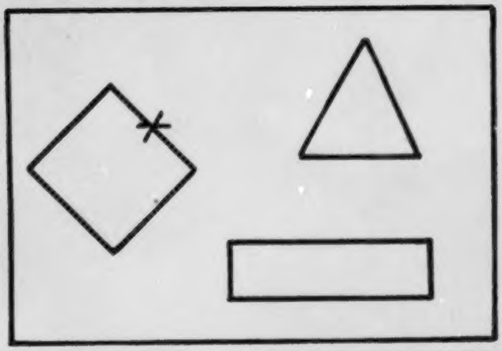
Ask children if it would be possible to use what you have drawn to make a model of a *rectangle* that is *not* a square.

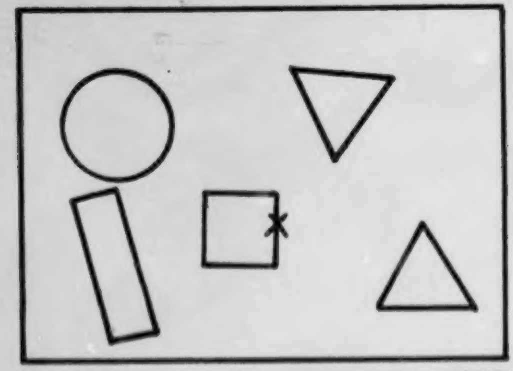
Now put on the chalkboard a drawing like this

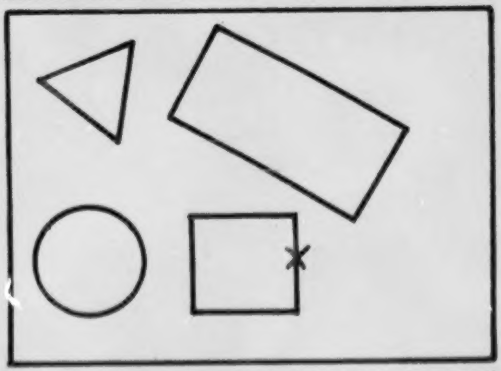


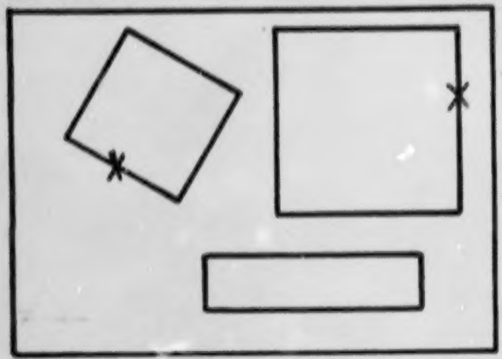
where the segments are not congruent. Ask children if it would be possible to use these to make a model of a square.

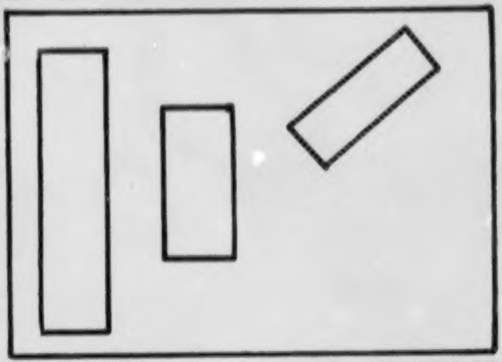


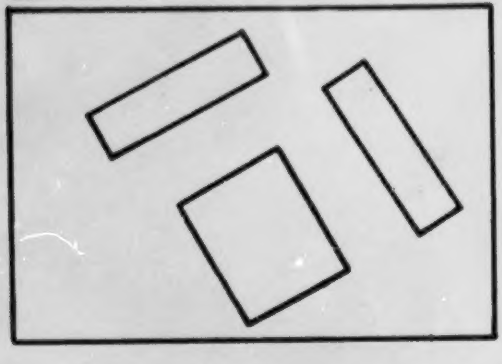


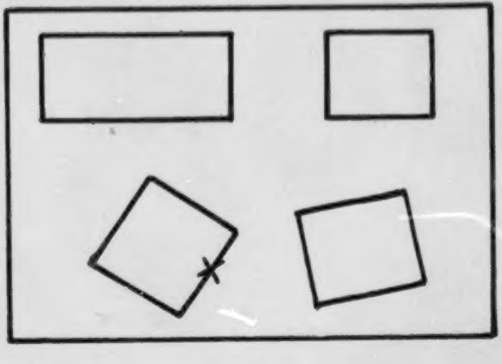


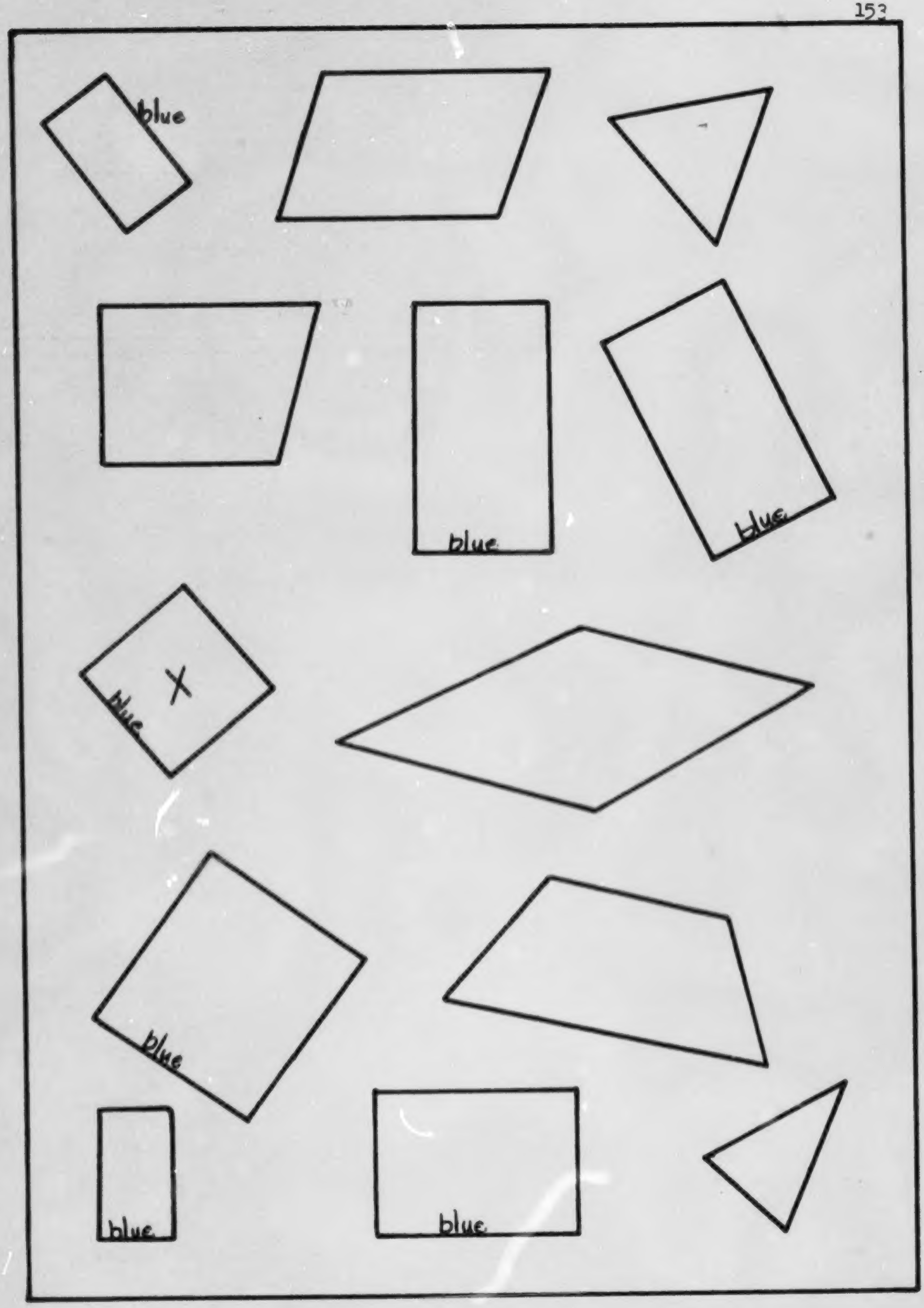








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VI-3. Fitting different shapes

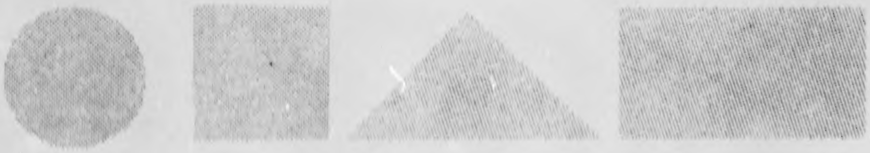
Objective: To distinguish different shapes by seeking to fit models of various regions

Vocabulary: Match, fit

Materials: Flannel board models of different sized and shaped regions. There should be two sets of congruent shapes, of contrasting colors (red and green for instance); also, at least one square model of larger size
Construction paper models of regions in two colors, as above, a set for each child

Suggested Procedure:

Place on the flannel board some of the red models as shown:



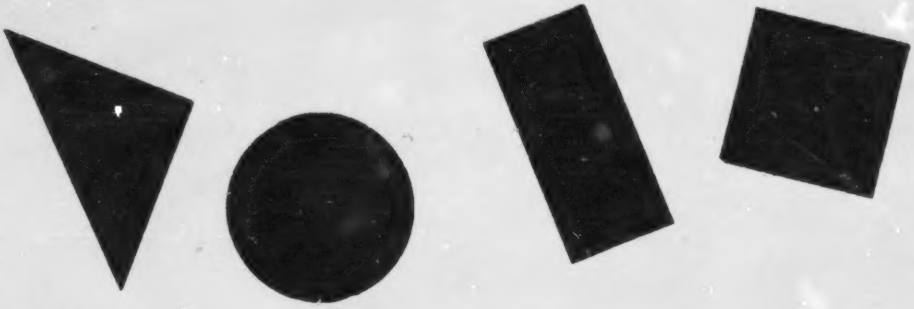
Talk about what it means to fit exactly, or to match exactly. Show how edges of coins of like denomination match exactly; discuss the way the edges of slices of bread often fit exactly in a loaf. Pages in a book match, and one end of an unsharpened pencil may fit exactly against the end of another, with nothing left over of either pencil.

Hold up a green rectangular shape which will fit exactly one of the red ones on the flannel board. Hold it with its edges parallel to the edges of the one on the board. Have it described as a rectangular region. Ask whether this region will exactly fit any of those on the flannel board. Have a child do the matching, and show that all edges match, or fit.

Remove the two shapes, place the green one on the flannel board, and ask whether the red one can be matched to it. Remove the green model and ask whether it would fit on any of the other shapes on the flannel board. Have a child try to fit it, and show clearly that there are some parts not covered up either on the red or on the green model.

Match the other green models to the appropriate red models in the same way, having them described each time as a model of a __________ region.

Without letting the class see what you are doing, arrange the green models on the flannel board in different positions.



Hold up the red triangular model in the same position it was in when it was first matched. Ask with what kind of region it might be matched. Turn the flannel board so that the children can see the shapes on it, then ask the children whether the red model region can be matched to one of those on the board. Caution them to be careful, for even this simple arrangement can cause difficulty for children who expect to see regions in positions with one edge parallel to the floor. Continue fitting the other models.

Hold up the square model of larger size and ask whether it could be matched to any of those on the board. Discuss the fact that a region must not only be the same shape but also the same size in order to fit exactly.

Distribute the sets of construction paper models. Have children place red models on one side of desk in patterns similar to those on the flannel board. Have them place green models on opposite side of desk.

Have children match circular regions, then other regions. If they experience little difficulty, give each child a red square region larger than those he has. Let children experiment to see whether this will fit any of the others.

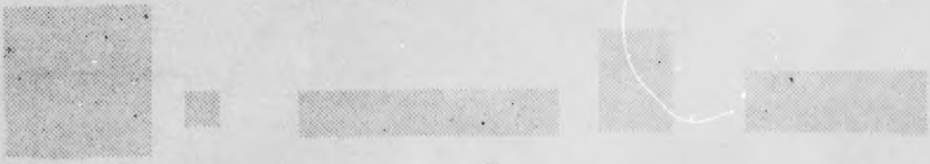
Geometric insights involved in this lesson include:

1. An awareness that regions of different shapes cannot be fitted one on the other
2. An awareness that two regions, even of the same kind, cannot be made to fit if they differ in size
3. An awareness that a model of a region can be rotated for possible fitting

An additional series of lessons can be developed to refine comparisons by fitting. A long thin rectangular region can be included with one that is nearly a square.

All the red shapes might be rectangular regions (including some square ones) such as two different sized square regions and three or more rectangular regions of differing dimensions and proportions.

The green regions should include all of these shapes as well as other rectangular (and square) regions of different proportions, including some like the following for which one pair of sides fits somewhere above but the other does not.



This time the pupil will need to recognize that, for the fitting, the lengths of opposite edges must be the same. By choosing some of the green shapes quite similar to the red ones, but not actually the same, good practice can be developed in estimating relative lengths. In most cases here, it would be profitable, before any attempted fitting is made, to discuss whether a given green shape will fit and what would be reasonable places to try it.

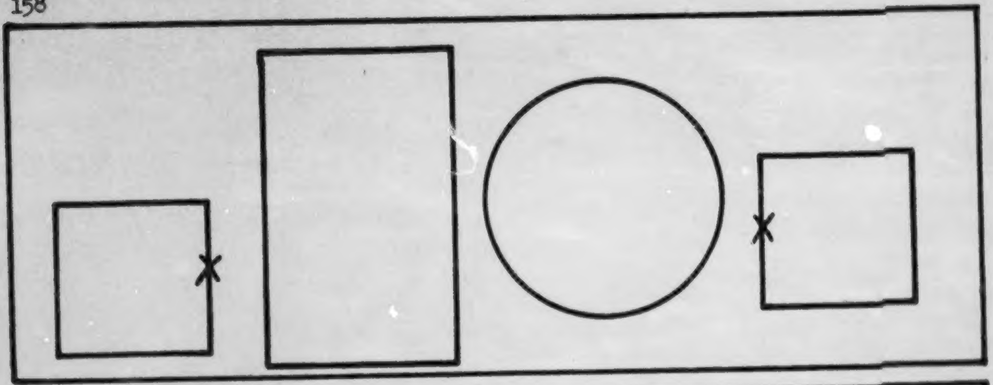
Another time some of the red shapes on the board should be turned in different positions. It is important to plan specifically for such a lesson.

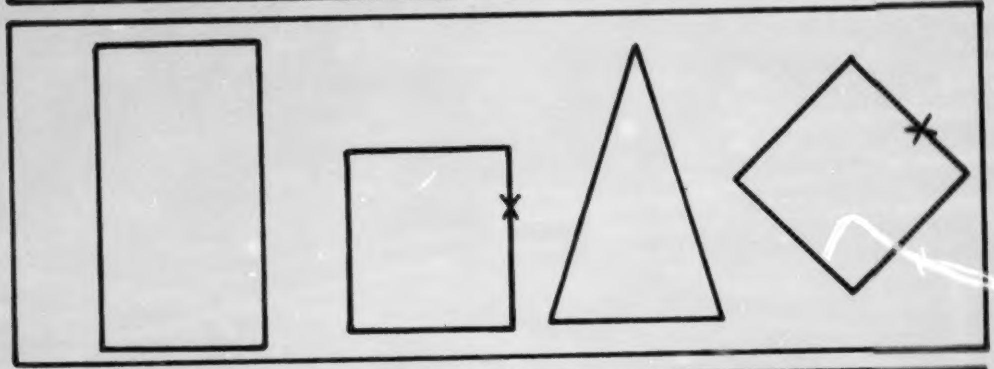
Geometric insights of these additional lessons would include:

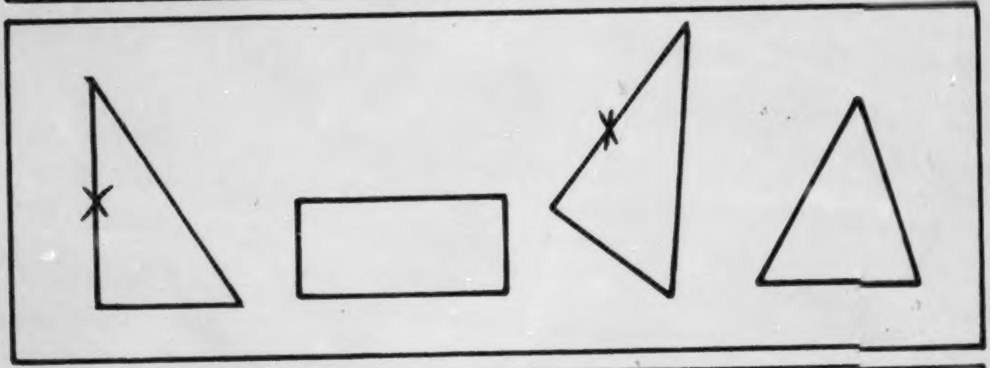
1. An awareness of the impossibility of matching a long, thin rectangle with one which is nearly a square
2. An awareness of the possibility of comparing visually the sides of a model to see whether they are likely to fit
3. An awareness of the possibility of rotating a model to make it fit another
4. The recognition of a rectangular or triangular region which does not have an edge parallel to the floor

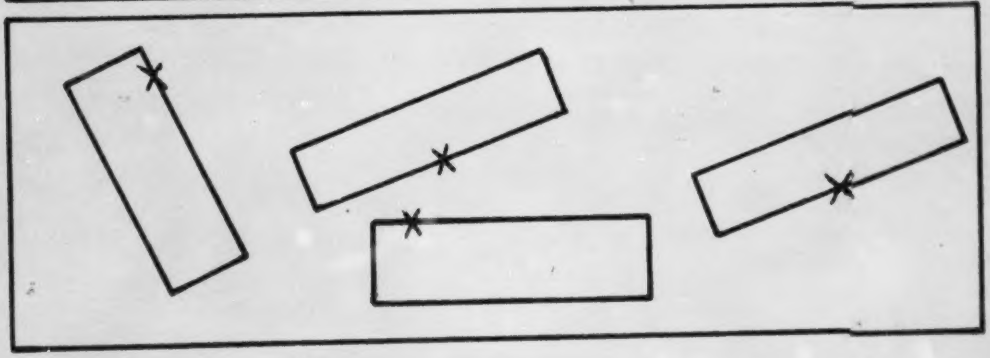
Pupil Page 38: Use this page to evaluate the ability of children to see fittings when figures are in different positions. The bottom set of figures has rectangles each of which can be matched to any other. Instruct the children to put a mark on the pictures of shapes that can be matched in each section.

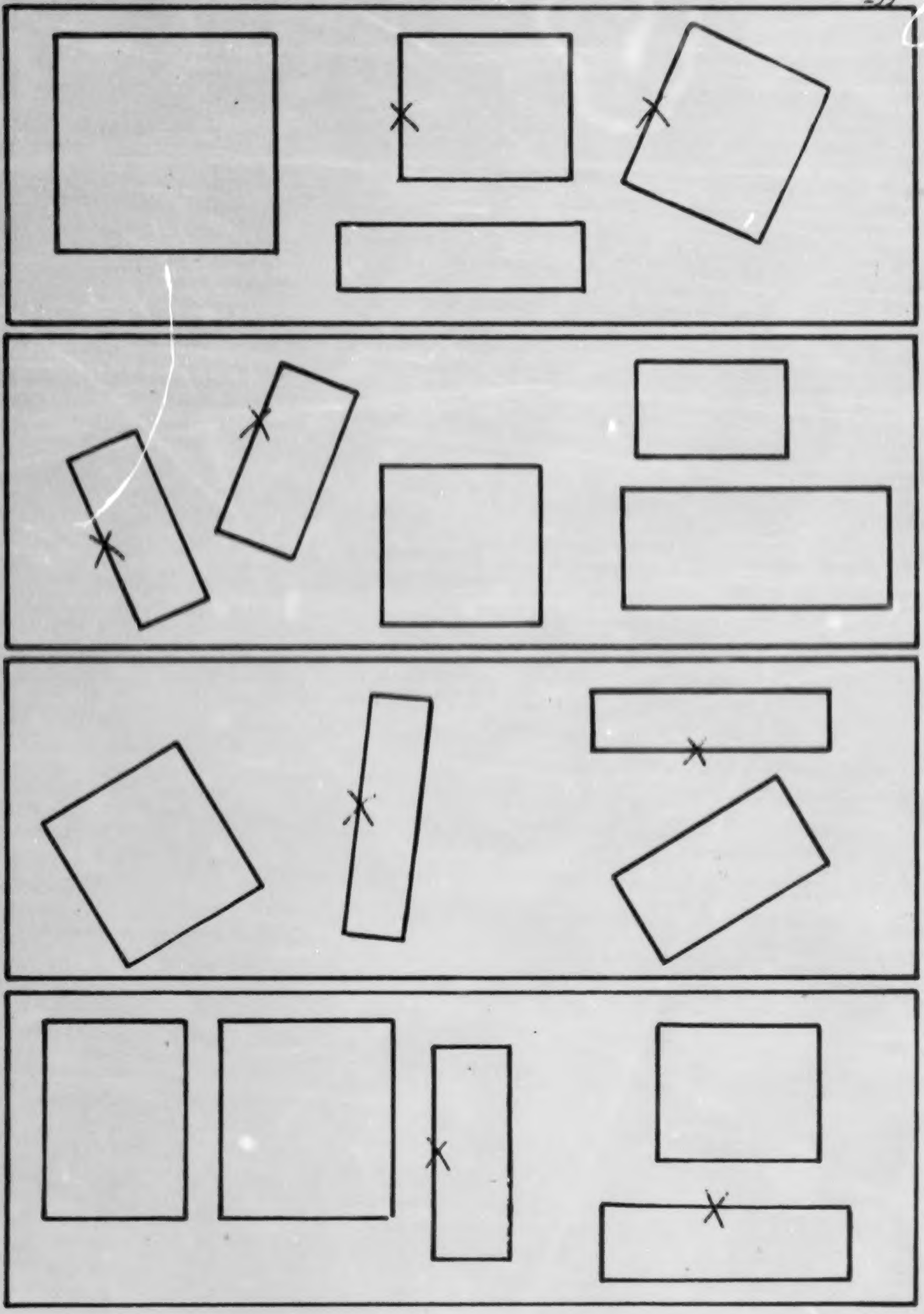
Pupil Page 39: Have children mark the two shapes that can be matched in each section.











Chapter VII

ADDITION AND SUBTRACTION

Background

The teacher may want to read again the Background for Chapter IV where several of the ideas of the present chapter are introduced informally. For easier reference the discussion here has been broken down into sections as follows:

1. Addition
2. Subtraction
3. Doing and undoing
4. Related number facts
5. Number names
6. Equations and equality
7. Addition and subtraction on the number line

1. Addition. We recall that addition is explained in terms of the union of two disjoint sets (that is, two sets which have no members in common). The number of members in the union of two such sets is the sum of the numbers of members of the two sets.

For example, to arrive (ultimately, not all at once!) at the equation $2 + 3 = 5$, we may begin with a two-member set consisting of, say, the two children John and Mary and join to it the three-member set consisting of Tom, Sue, and Jane. These two sets are disjoint, because they have no members in common. Their union is the five-member set consisting of John, Mary, Tom, Sue, and Jane.

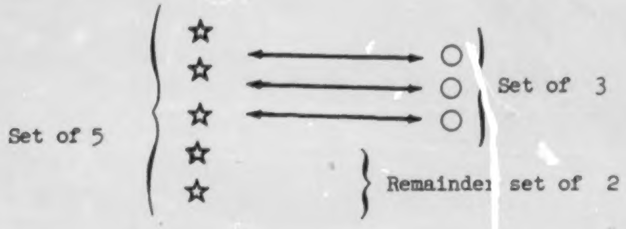
(If two sets are not disjoint, we can see that the number of members in their union is less than the sum of the numbers of members of the two sets. For example, suppose we are given the two-member set consisting of John and Mary, and the three-member set consisting of Mary, Sue, and Jane. These two sets are not disjoint, because they have in common the member Mary. The union of these two sets is the four-member set consisting of

John, Mary, Sue, and Jane. In this union the number of members, 4, is less than [rather than equal to] the sum of the numbers, 2 and 3, of members of the two given sets.)

Historically there seems to be little doubt that the joining of sets (sets of sheep, sets of shekels, etc.) constituted the original motivation for the introduction of addition. In any case, joining of sets is the natural approach to addition, and is easily understood by children. It also serves to motivate so naturally the commutative property of addition (3 + 2 = 2 + 3, etc.) that this property can profitably be noted (at least orally) at the outset. Here the difficulty may be to convince children that commutativity of joining is worth noting at all. But of course this underlies the commutativity of addition; and it is not obvious to the children that 3 + 2 equals 2 + 3.

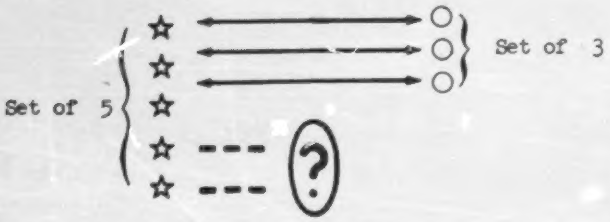
2. Subtraction. Just as joining two sets offers a natural approach to addition, so removing a subset from a set offers a natural approach to subtraction. (These devices are used together to convey the inverse relationship of addition and subtraction of a given number. See Section 3.) Thus, to arrive (ultimately) at the equation 5 - 3 = 2, we may start with a set of five objects, remove a subset of three of them, and note that there remains a subset of two.

The comparing of two sets by means of one-to-one correspondence also leads to subtraction of numbers. Consider, for instance, a set of five stars and a set of three balls. When we compare these sets by one-to-one correspondence, as shown below,



we find that there is a remainder set of 2.

We can also describe subtraction in terms of how many must be joined to a given set to obtain an equivalent set, as shown schematically below.



This leads to the so-called "missing addend" approach. We can see that 5 - 3 is the answer to the question: 3 + ? = 5.

3. Doing and undoing. We have been talking about two operations on sets, joining and removing. There is a sense in which the joining and removing of a given set are inverse operations. Each of these operations "undoes" what the other does. If we start with a set, join to it a given set (disjoint from it), and then remove from the resulting union this same given set, we get back the set we started with.

For instance, suppose we start with the set whose members are John, Mary, and Tom and join to it the set whose members are Sue and Jane. We thus get, as the union of these two sets, the set consisting of John, Mary, Tom, Sue, and Jane. Suppose we now remove from this union the set that was joined, that is, the set whose members are Sue and Jane. In this way we get back the set we started with, namely the set consisting of John, Mary, and Tom.

Instead of first joining and then removing, we can first remove and then join. Suppose that we start with a set and remove from it a given subset, getting a certain remainder set. If we then join to this remainder set the same given set, we again get back the set we started with.

The inverse relationship between joining and removing a given set is reflected in the inverse relationship between adding and subtracting a given number. In both cases each operation "undoes" what the other does.

In fact, when we focus attention on the numbers of members involved, the sets we have just used to illustrate the inverse relationship between joining and removing a given set also show the inverse relationship between adding and subtracting a given number. Thus when we take the number 3, add to it the number 2, and then subtract from the sum this same number 2, we get back the number 3 we started with. We say that the operation of adding 2 has as its inverse the operation of subtracting 2. More generally, the operation of adding any given number has as its inverse the operation of subtracting that same number.

By thinking of the example where we first removed and then joined a given set, we similarly see that by starting with a number, first subtracting from it a given number, and then adding to the result this same given number, we again get back the number we started with. We say that the operation of subtracting a given number has as its inverse the operation of adding this same number.

4. Related number facts. It is important that the number facts be presented to children, not as a discrete sequence of individual items, but in related groups.

For instance, a set of 4 objects may be partitioned into a set of 3 objects and a set of 1 object. This is just one partition, but it leads (in ways already noted) to four distinct number facts:

$$3 + 1 = 4$$
$$4 - 1 = 3$$
$$1 + 3 = 4$$
$$4 - 3 = 1$$

If these four facts are presented to children as all stemming from this same partition, they are surely more meaningful than if they are presented simply as isolated facts to be memorized.

There are other ways of grouping number facts into related families. For example,

$0 + 4 = 4$
$1 + 3 = 4$
$2 + 2 = 4$
$3 + 1 = 4$
$4 + 0 = 4$

form a set of related number facts. The common feature here is, of course, that these facts all arise from partitioning a set of 4 objects: into a set of 0 and a set of 4; into a set of 1 and a set of 3; etc. The arithmetic result is that we obtain various expressions for the same number, 4, as a sum of two numbers. Again it is helpful to present these facts together as a related family.

5. Number names. Let us consider more closely the related number facts we have just been discussing:

$0 + 4 = 4$
$1 + 3 = 4$
$2 + 2 = 4$
$3 + 1 = 4$
$4 + 0 = 4$

The first of these may be thought of as saying:

> The number that results from taking 0 and adding to it 4 is the number 4.

The next says:

> The number that results from taking 1 and adding to it 3 is the number 4.

And so on.

From a slightly different point of view, we may say that we simply have here a variety of different ways of designating, or naming, the same number. We may call it

"the number 4,"

or

"the number that results from taking 0 and adding 4,"

or

"the number that results from taking 1 and adding 3,"

and so on. Written in our familiar brief notation, these different ways of naming this number appear thus:

$$\begin{gathered} 4 \\ 0 + 4 \\ 1 + 3 \\ 2 + 2 \\ 3 + 1 \\ 4 + 0. \end{gathered}$$

These are by no means all the ways of designating or naming the number 4. Roman numerals, written words, and subtraction yield more:

IV

four

$$\begin{gathered} 4 - 0 \\ 5 - 1 \\ 6 - 2 \end{gathered}$$

and so on. Later on in school children will encounter still others, like

$$\begin{gathered} 2 \times 2 \\ 4 \times 1 \\ (\tfrac{2}{5}) \times 10 \\ 8 \div 2 \\ (7 + 5) \div 3 \end{gathered}$$

and many more.

Thus *the same number may have many different names*. And this is just as true of every number as it is of the particular number, 4, that we happen to have been using as an example.

One more point. Sometimes we have to refer, even within the same sentence, to both a number and a symbol that serves as a name for that number. It is then often helpful to distinguish the number from its symbolic name by enclosing the name in quotation marks, thus:

The number 4 has "4" as one of its names.
The number 4 has "four" as one of its names.
The number 4 has "1 + 3" as one of its names.

6. Equations and equality. As we have just seen, the same number may have many different names. Thus, the symbols "1 + 3" and "4" are each names for the number 4. We symbolize this fact by writing

$$1 + 3 = 4.$$

In words we read this:

"The number 1 + 3 equals the number 4,"

or

"The number 1 + 3 is the same as the number 4,"

or, emphasizing the role of the symbols,

"The number named by the symbol '1 + 3' is the same as the number named by the symbol '4'."

In mathematics, "equals", correctly used, can always be replaced by "is the same as." (Thus it is incorrect if we call two different sets "equal" just because they have the same number of members, or if we call two different geometric figures "equal" just because they have the same size and shape!)

Some teachers may also wish to introduce the mathematical symbol for "is not equal to" as well as the symbol for equality. This symbol for inequality is obtained by drawing a line segment through the symbol for equality. Thus

$$2 + 2 = 4,$$

but

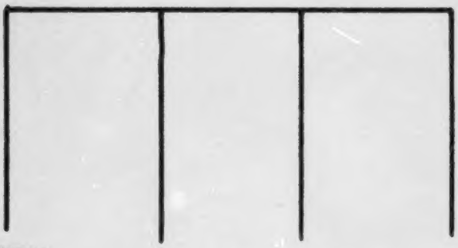
$$2 + 2 \neq 5.$$

VII-1. Review of joining: adding

Objective: To review joining sets
To use number in relation to this set experience
To use terms plus and equals orally in connection with number to make equations

Vocabulary: Add, plus, equals, number, equation

Materials: Sets of colored beads, shoestring, large sheets of paper divided into three columns for a chart



Suggested Procedure:

Ask a child to pick out a set of 2 red beads from a box of beads. Have the child place the beads on the string. After the beads are on the string, note that they make our first set. Explain to children that we will keep a record of our work and record this number in the first column of the chart.

Ask a child to pick out a set of 3 yellow beads. We want to join this set to the first set by putting the yellow beads on the string. An attempt should be made to get the whole set of yellow beads on the string so they can be joined to the red ones as a set rather than one at a time. The teacher should then explain that she will record the number of yellow beads in the second column of the chart.

Continue to point out to the children as the chart is developed that the left-hand column tells the number of objects in the set we started with and the middle column tells the number of objects in the set joined to the first set. Join the set of yellow beads to the set of red beads.

> How many members are in the set of red and yellow beads? (5)

Explain to the children that this number tells the number of members there are when the second set is joined to the first. The teacher should record this number in the right-hand column of the chart. Review with the children the reason for placing each of the numerals on the chart.

Start Set	Join Set	Union
2	3	5

> Look at the numerals used to make a record of our work with the sets of beads.
>
> We can use these numerals to make an equation.
>
> In the equation we say, "two plus three equals five."
> We are adding the number 3 to the number 2.
> The result is the number 5.

Make a line across the chart before continuing with other sets. Use no sets in which the sum is greater than five. It would be wise to use imagined sets as well as actual manipulative sets.

Pupil Page 40 : Ask the children to draw a line between the picture of the beads and the numbers they would use in an equation about these beads.

Further Activities:

After several experiences have been recorded, encourage the children to discover some of these relationships: (The chart given here is only a sample and no attempt should be made by the teacher to develop the same chart in her class. However, the relationships will be the same. The letters are used here only to help the teacher identify the relationships which have been illustrated on this chart.)

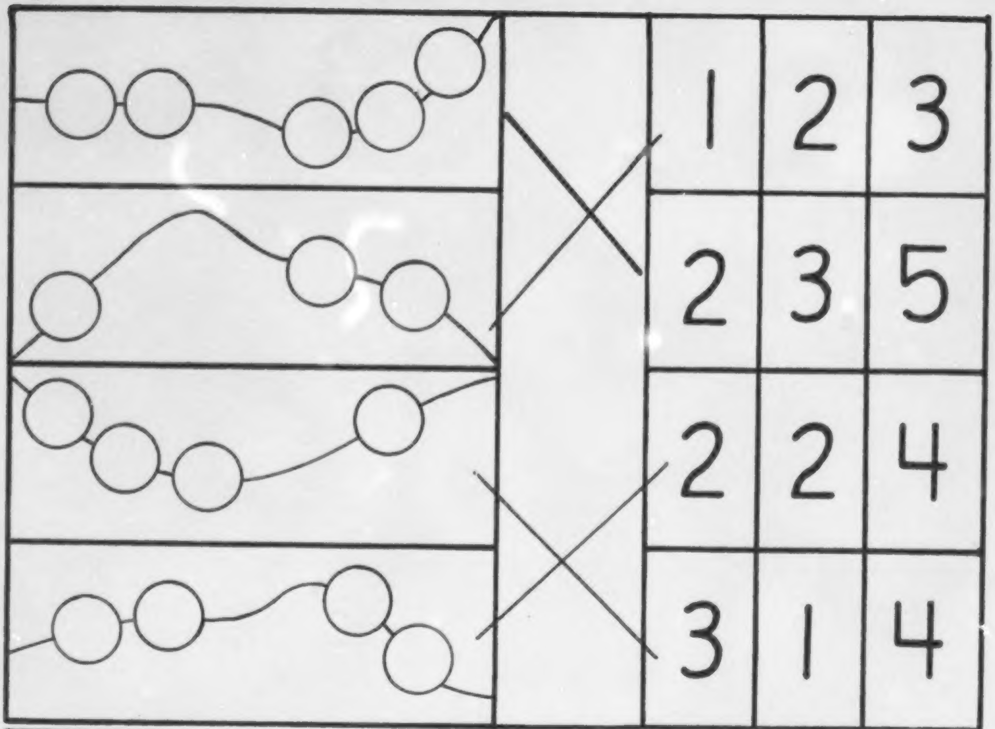
a	2	3	5
b	1	2	3
c	2	0	2
d	1	1	2
e	0	4	4
f	2	1	3
g	4	0	4

Sometimes we used the same numbers, but they did not come in the same order (b and f, e and g).

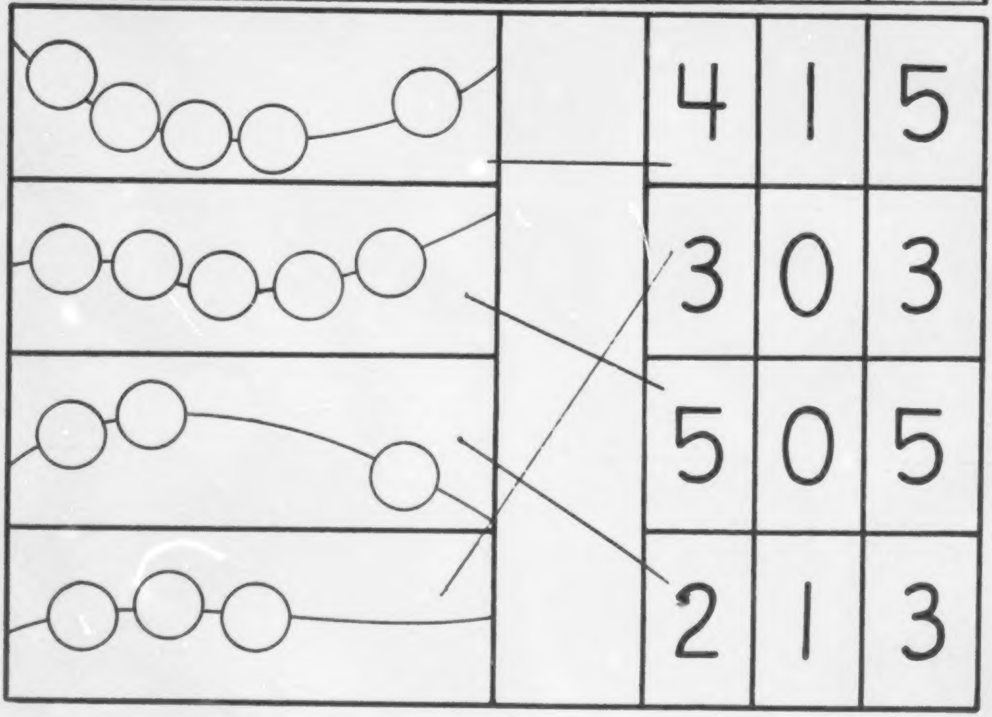
Sometimes the number for the union of the sets was the same as the number of members in one of the sets (c, e, g).

Sometimes the number for the union of the sets was greater than either of the other numbers (a, b, d, f).

The number for the union of the sets was never less than either of the other numbers.



1	2	3
2	3	5
2	2	4
3	1	4



4	1	5
3	0	3
5	0	5
2	1	3

VII-2. Review of removing: subtracting

Objective: To review the idea of removing a subset
To use number in relation to this set experience

Vocabulary: Minus, subtract

Materials: Set of flannel cutouts or set objects
Set of numeral cards 0, 1, 2, 3, 4, 5
Chart holders to display numeral cards

Suggested Procedure:

Ask a child to form a set with 5 objects on the flannel board. Display the set of numerals and ask if a child can find the card for the number of members in the set. Place the card on the left at the bottom of the flannel board. Ask a child to describe a subset of the set. Remove the subset and place it on the flannel board apart from the original set.

> How many members are in the set that was removed? (2)

Ask the child to find the numeral card which tells this number and place it on the flannel board at the right of the first card.

> What do we call the set that is here (indicate the part that was not moved)? (The remainder set)
>
> How many members are in the remainder set? (3)

Ask a child to find the numeral card for the number of members in the remainder set and place it on the flannel board at the right of the other cards.

> The numerals at the bottom of the flannel board can be used to make an equation. We say 5 minus 2 equals 3.

We are subtracting the number 2 from the number 5.
The result is the number 3.

As a review it might be wise to ask the children the following questions:

Which set has 5 members? (The set we started with.)
Which set has 2 members? (The set we removed.)
Which set has 3 members? (The remainder set.)
What is the equation? (5 minus 2 equals 3)

In further development of this lesson the numerals should be recorded on a chart similar to the one used in Lesson 1. The order in which they are recorded in this lesson should be followed on the chart. It is important, too, that the children recall after each experience the number used to describe each set. The column on the left is for the number of members in the starting set; the column in the middle is for the number of members in the set that was removed; and the column on the right is for the number of members in the remainder set.

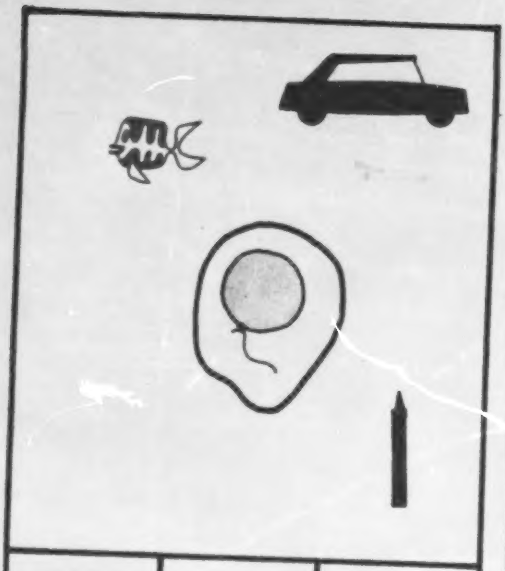
Continue to work with the concept by using other sets. At this time the sets should not have more than 5 members. The completed chart would look like this:

5	2	3
4	1	3
2	2	0
3	1	2

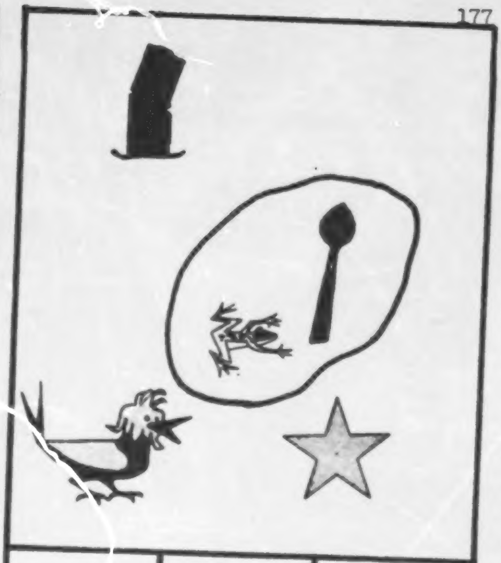
etc.

Pupil Page 41: Each box pictures a set in which a ring has been made to represent a subset. The children are to imagine that this subset has been removed from the set and are to record the number of members in the remainder set in the space provided.

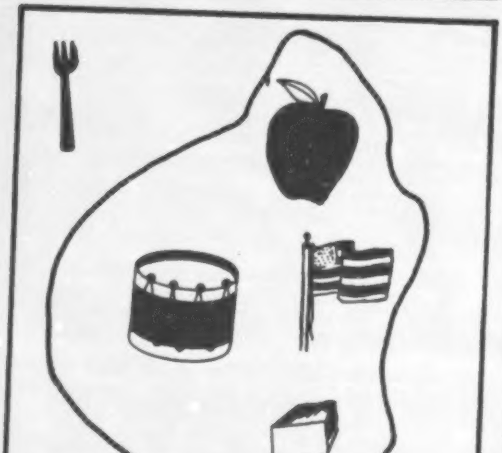
Pupil Page 42: Direct the children to count the number of members of the set. They are to indicate a subset to be removed by making a ring around it. They are to write the numerals which would be used to make a record. (In this lesson all children's charts will not necessarily be the same.)



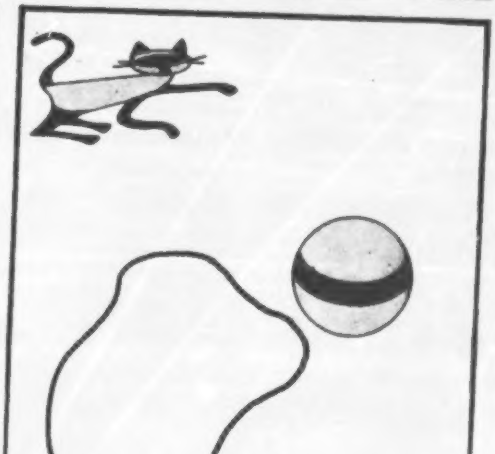
4	1	3



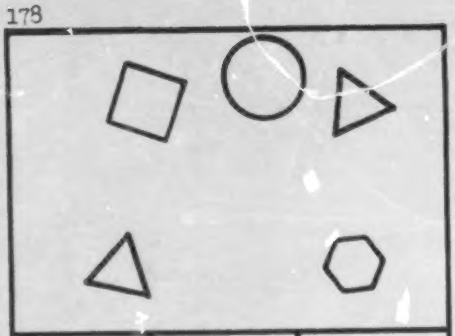
5	2	3



5	4	1

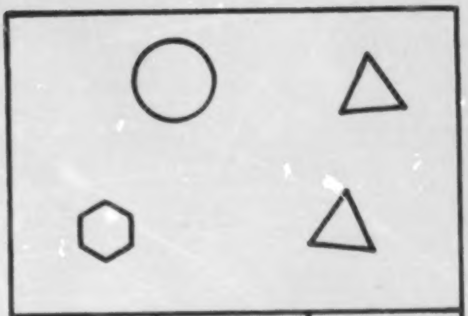


2	0	2



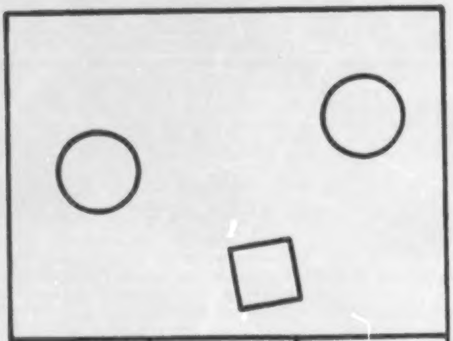
5

answers will vary



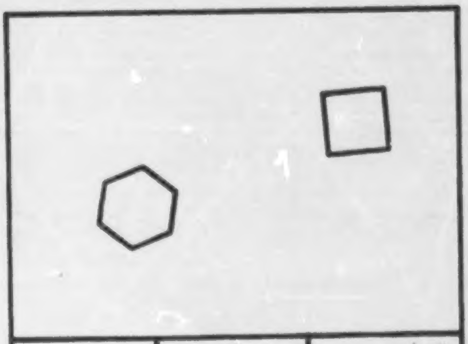
4

answers will vary



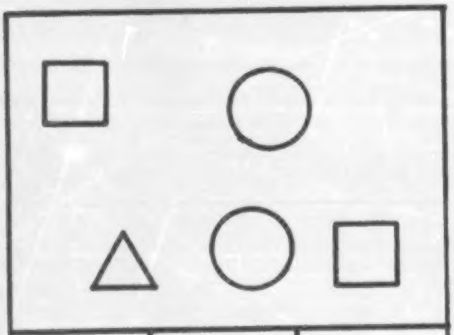
3

answers will vary



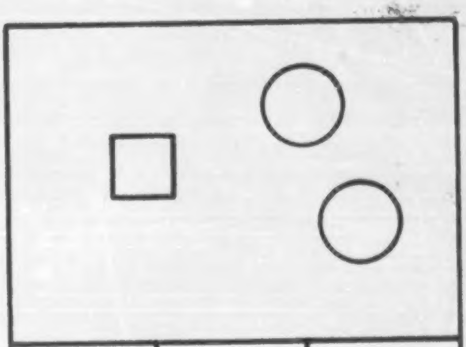
2

answers will vary



5

answers will vary



3

answers will vary

VII-3. Review and extension of partitioning

Objective: To review partitioning of sets
To use number in relation to this set experience
To show various partitionings of the same set

Vocabulary: No new terms

Materials: Flannel board and flannel cutouts, chart paper to record work, piece of yarn or strip of paper

Suggested Procedure:

Start with a set of 4 objects on the flannel board and tell the children they are going to partition this set and make a record of their work.

Write the 4 in the right-hand column of the chart. This names the number of members in the set that will be partitioned. Ask a child to partition the set into two subsets. Use a strip of paper or piece of yarn to make the partition. Discuss with the children the number of members in each of the subsets formed by the partition. This can be summarized by saying that the set with 4 members was partitioned into a set with 3 members and a set with 1 member. Complete the chart by writing 3 and 1 on the chart in the middle and left-hand columns.

4	3	1

Develop with the children other ways in which the set of 4 objects can be partitioned. Explore all of the possibilities and record them on the chart. If a child partitions the set in a way that has already been done, see if the child can find the place on the chart where this partition has been recorded.

Help the children to see the relationship between some of the set partitions (3 and 1; 1 and 3). It may be desirable to note these ideas in a lesson other than the one in which the chart is developed, unless it arises naturally at this time.

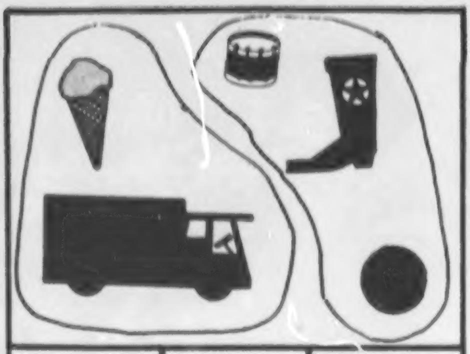
Help children to see that if after making a partition we remove one of the subsets, then our remainder set is the other subset of the partition. By discussion also note that if, after the partition is made, the two sets are joined, then the union of the sets results in the original set.

Charts should also be developed with sets which have 1, 2, 3, and 5 members.

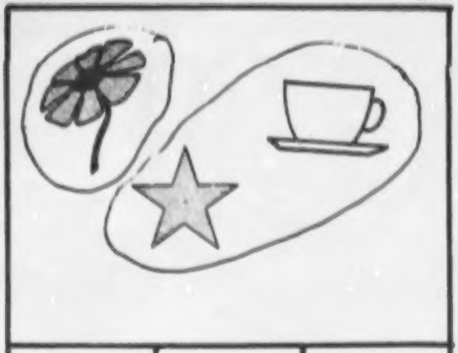
As a review, some children might make charts independently. This could be done with three columns on half sheets of paper.

Pupil Page 43 : Direct children to count the set in each box and record the number in the space at the bottom of the box. Then they are to partition the set by ringing two subsets whose numbers are those given in the chart.

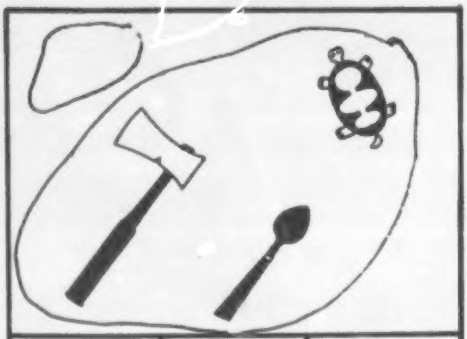
Pupil Page 44: For each set pictured, ask children to count the members in the set and write the numeral in the first box. Then they are to make a partition of the set and complete the chart by recording the partition. Different partitions should be made when the original sets are equivalent.



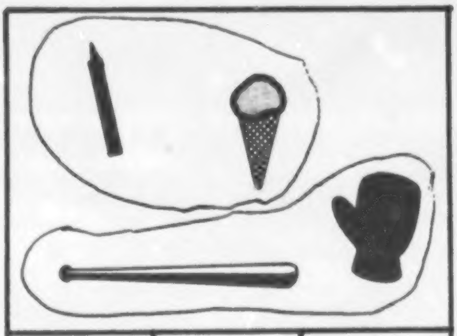
5	2	3



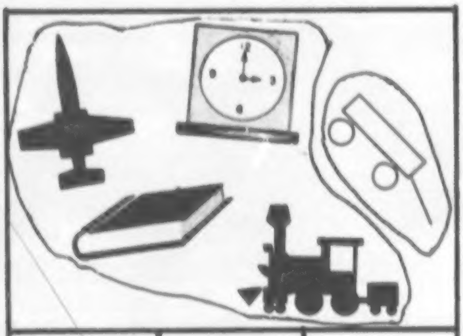
3	1	2



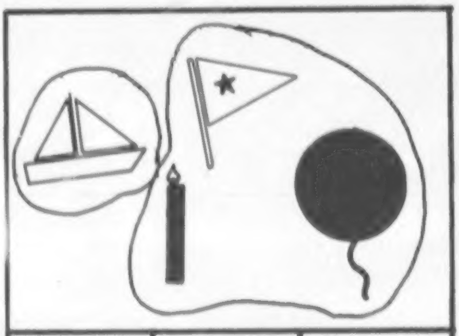
3	3	0



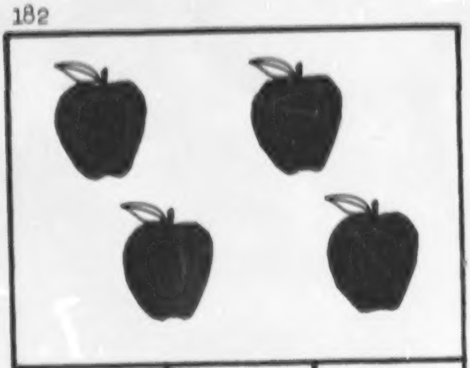
4	2	2



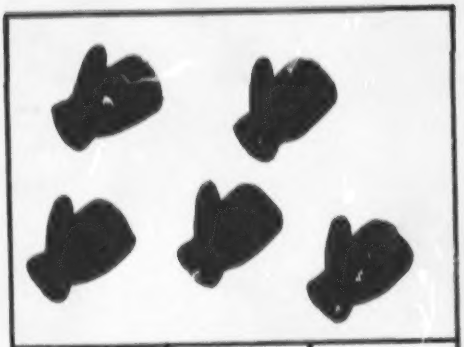
5	4	1



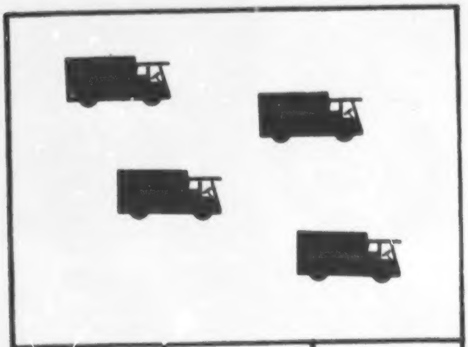
4	1	3



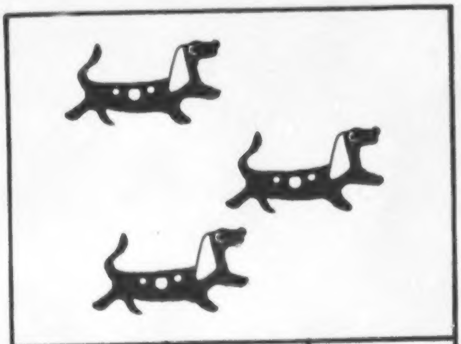
4	answers will vary	



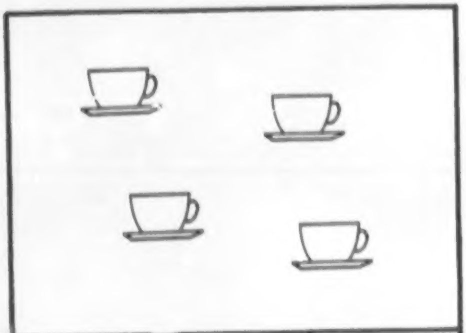
5	answers will vary	



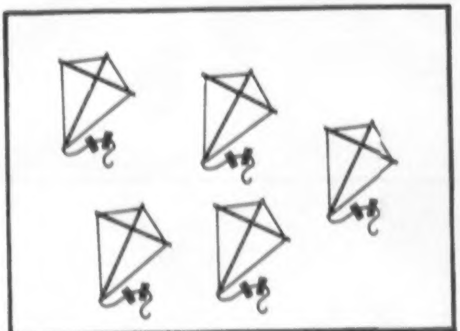
4	answers will vary	



3	answers will vary	



4	answers will vary	



5	answers will vary	

VII-4. Review and extension of comparing

Objective: To review comparing of sets
To use number with this set experience

Vocabulary: Greater than, less than

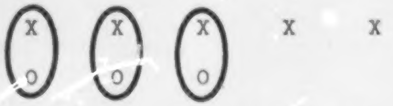
Materials: Flannel board and flannel material, suggested: apples and lemons
Two sets of numeral cards, 0, 1, 2, 3, 4, 5
(Sandpaper or flannel strips on the back of the card will make it stay on the flannel board.)

Suggested Procedure:

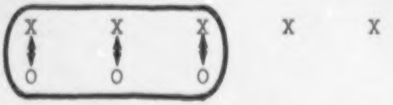
Ask a child to place a set of 3 apples on the flannel board. Ask another child to place a set of 5 lemons on the flannel board. Display the set of numeral cards and ask a child to place the correct card on the flannel board with each set, noting that we can see which set has more or fewer members by pairing the members of the sets. Ask a child to do this.

The teacher will want to be aware of the following ways in which children will compare the sets:

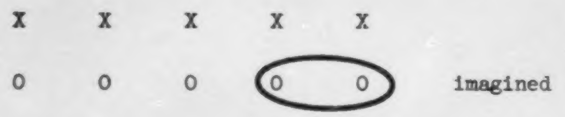
1) The child may place the sets in rows and make a one-to-one correspondence. After this is done he may count the members that aren't paired.



2) The child may remove a set equivalent to the set with fewer members from the set with more members and count the remainder set.



3. The children may place the sets in rows and then note how many more would be needed to make the sets match.



After the work with the sets is completed and the children have found how many more members are in the set of lemons, place the numeral card on the flannel board. The number of the set with more members is greater than the number of the set with fewer members; 5 is greater than 3. We say that the number of the set with fewer members is less than the other number: 3 is less than 5.

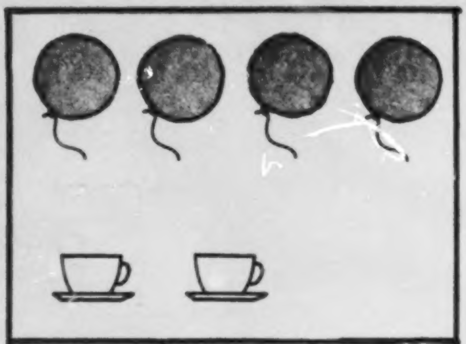
Pupil Page 45: Each of the six boxes on the page pictures two sets, with the number of members of each recorded below. Pupils are to record the difference of these numbers in the space provided.

Pupil Page 46: Pupils are to compare the sets and write the numeral in the space provided that indicates how many more are needed so the sets match.

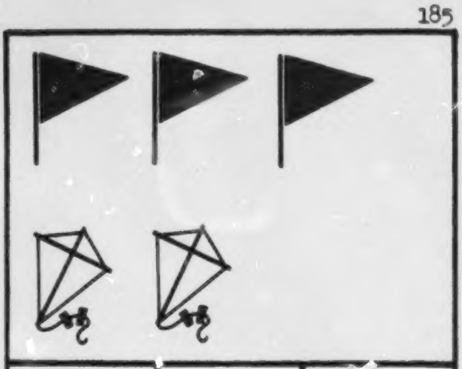
Further Activities:

Develop a chart with the children in which the numerals are recorded in this way.

The left hand column is used to record how many members are in the set with more members. The middle column is used to record how many members are in the set with fewer members. The right hand column is used to record either how many more or how many fewer members are in one set than in the other. It is significant to point out as the work progresses that this number is the same, whether it indicates how many more or how many fewer.



4	2	2



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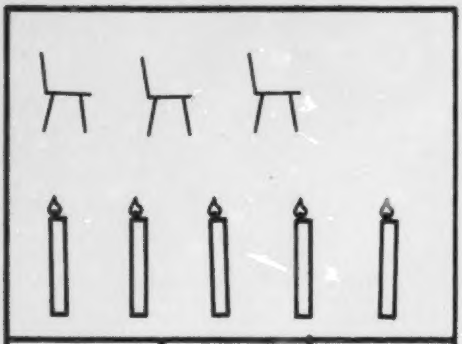

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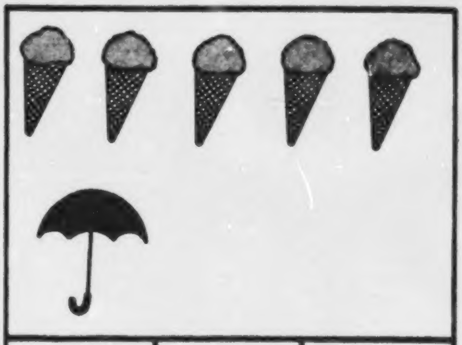

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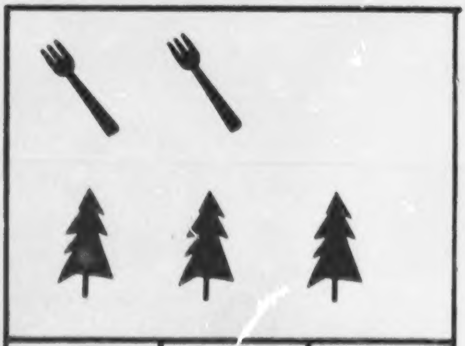

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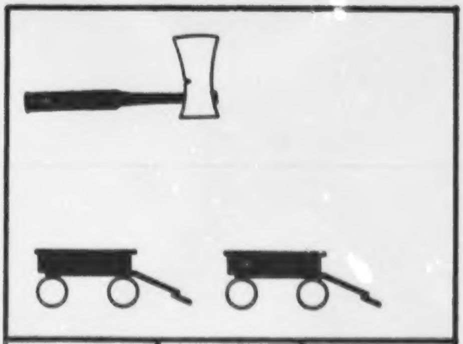

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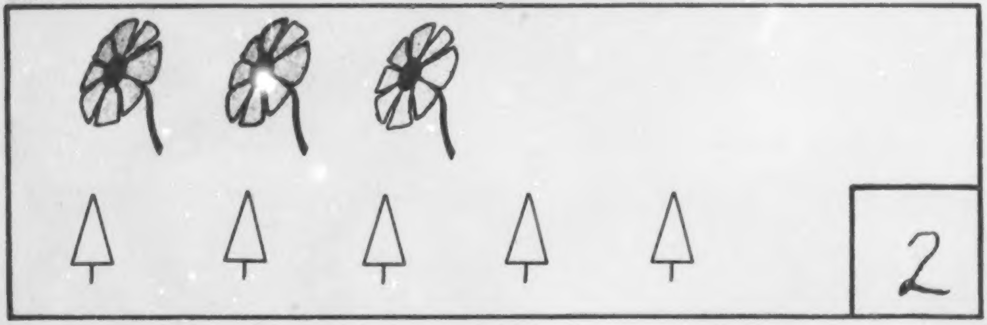

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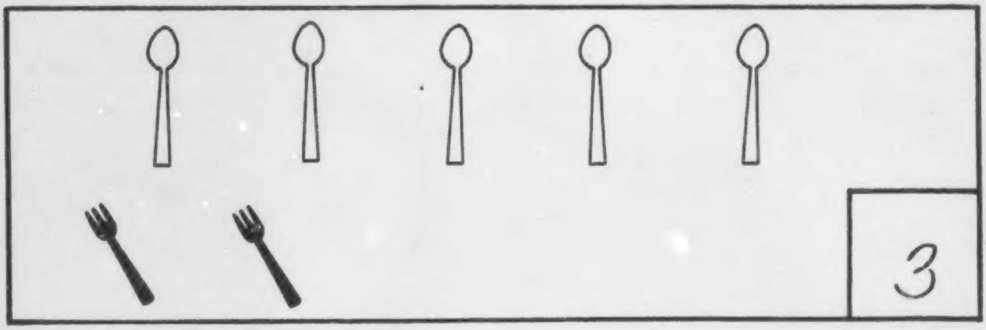

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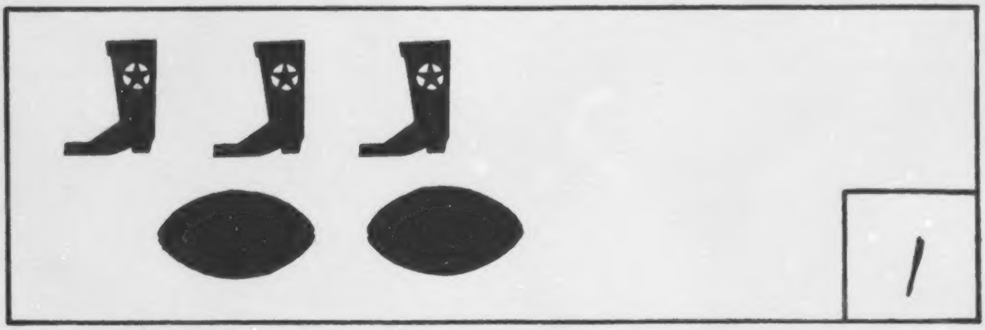

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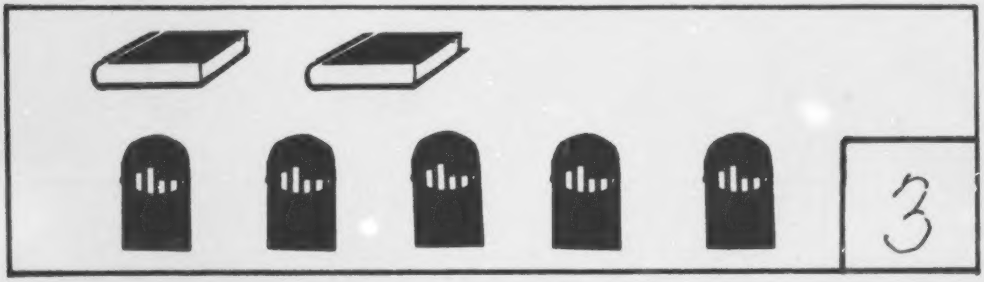


3

VII-5. Writing equations

Objective: To review comparing sets, joining sets and removing sets

Vocabulary: No new terms

Materials: Chart holders

Numeral cards 0, 1, 2, 3 4, 5

Suggested Procedure:

The lesson should consist of many imagined and manipulative experiences with sets to develop the children's ability to record the numerals used in connection with a set experience. These will be recorded by using numeral cards and a chart holder in this lesson. If the problem leads to an equation, as in adding and subtracting, this should be presented. Each type of problem will be illustrated once here with the understanding that the teacher will present others of each type. She will want to use illustrations in comparing sets where children are asked "how many less" as well as "how many more."

Joining sets:

Jerry was playing with his 3 boats in the pond. John came up with his boat and put it in the water too. How many boats were in the water while Jerry and John were both playing?

Some of the children may know the answer immediately but the purpose here is to strengthen their understanding and ability to place the numerals in the right order in making the record. It is also to review their understanding of what each numeral represents.

What was done with Jerry and John's sets of boats?
(They were joined.)

What do we do with the numbers which describe the sets that have been joined? (Add)

Which numeral will we place first in the chart holder? (The one that names the number of things we started with. Jerry's set.) (3)

Ask a child to place the numeral card for this set in the chart holder. Start at the left.

Which numeral will we place in the holder next? (The one for the set of boats that was joined to Jerry's boats.) (1)

We said that we would add the numbers if we had joined the sets. How many do we have if we add 1 to 3 ? (4)

Place the numeral card for 4 in the chart. Review the place of each numeral card in the chart. Ask a child what equation is used with these numerals. (Three plus one equals four.)

Comparing sets:

Mary has three dolls. She has four stuffed toy animals. How many more stuffed animals than dolls does she have?

Once again the children may know the answer, but it is our purpose to help them record the numerals that have been used in connection with the set experience. In recording this problem they should first put the numeral card for the set with more members on the chart. The next numeral card is the number of members in the set with fewer members. Then place the numeral which indicates how many more members one set contains than the other; or how much greater one number is than the other.

Removing sets:

Tommy's mother gave him five balloons. He gave one to his friend Bill. How many balloons did Tommy keep for himself?

In recording this, which numeral do we put on the chart first? (The numeral for the set Tommy had to start with.) (5)

Place this numeral card on the chart.

What happened to Tommy's set of balloons? (He gave one to Bill. One member was removed.)

Place this numeral card on the chart.

When we think of Tommy's set we have to remove one member. When we work with the numbers we will subtract 1 from 5. Who can tell us this number? (4)

Place the numeral card, 4, on the chart.

What equation tells us we have subtracted one from five? (Five minus one equals four.)

VII-6. Introduction of symbols +, - and =

Objective: To introduce the meaning and use of the symbols -, + and =

Vocabulary: No new terms

Materials: Chalkboard, set materials or flannel board materials

Suggested Procedure:

At this time children should be familiar with the words plus, minus, and equals because they will have used an oral equation to describe the adding and subtracting of numbers associated with the joining and removing of sets. They will be familiar with the order in which the numerals are recorded in an equation because of the development of the charts in the preceding lessons. Tell the children that they will learn some new symbols to be used in writing equations. Tell them that the symbol we write for plus is +, the symbol for minus is -, and the symbol for equals is =.

Present the symbols on cards or on the chalkboard as they are introduced. Review the names for the symbols until you feel that the children know them.

The children will have used charts for recording the numerals in connection with comparing sets and partitioning sets as well as with joining sets and removing sets. But at this time the new symbols shall be employed only in equations arising from the joining and removing of sets.

Present actual set experiences in which addition is needed. Place 2 flannel board objects on the flannel board and join a set of 4 objects to it. Ask the children to determine which numbers are used to describe the set experience. Write these numerals on the chalkboard. Discuss with the children the order in which the numerals are placed. Ask the children for the equation which would be used with these numerals, in relation to this experience. Place the appropriate symbols between the numerals and read the equation. Call to the children's attention that the order is still the same and that the reason each numeral is in its place is still the same.

Present imagined set experiences in which addition is used.

> Janet has 3 pencils in her desk. She is going to buy 3 more pencils at the store. When she takes the 3 new ones to school, how many will she have in her desk?

Ask the children what numerals are needed to describe the sets. Write the numerals on the board. Ask them what equation would be used with these numerals. Place the symbols between the numerals and read the equation.

Discuss with the children the similarity between the imagined set experience about the pencils and the written equation. The first 3 tells the number of pencils Janet had in her desk, the second 3 tells the number she added to the first set and the missing number is the number that tells how many she had when she joined the sets.

After writing equations about actual set experiences and imagined experiences, equations should be presented and experiences related to them.

Present the equation $3 + 1 = \boxed{}$. Ask a child to read the equation. (The phrase, what number, should be used when the box is shown.)

Some children will know immediately the number that is needed to complete the equation. Those who do not should learn to use either imagined or actual set experiences to help find the missing number.

Use flannel board material and ask a child to place on the flannel board the first set which is described in the equation. Establish what is to be done to the set by discussing the symbols used in the equation. Join a set with one member to the set with three members.

> What number tells how many are in the set? (4)
> This is the number that completes the equation.

Ask a child to write 4 in the box. Complete several equations using actual set experiences.

Write the equation $2 + 3 = \boxed{}$. Ask a child to read the equation. Tell the children to think of sets which the numbers in the equation describe. Ask a child to tell about the sets, e.g., I have two red boats and three blue boats, if I put the blue boats on the shelf with the red ones, how many do I have? Discuss with the children what was done with the boats we were thinking about. (They were joined or placed on the shelf together.)

> What number tells how many boats we are thinking about?
> (5)
>
> This is the number that completes the equation.

Ask a child to write 5 in the box. Complete several equations using imagined set experiences.

In connection with subtraction, present both actual set experiences and imagined set experiences to introduce the equation. This activity should be followed by relating set experiences to an equation in order to complete it. The procedure would be the same as that used for addition. (Remember at this time to use the subtraction equation only with removing of a set.)

It is possible that an entire class period may need to be devoted to the plus symbol, deferring to the next mathematics class period the use of the minus symbol. After use of both forms of equations is well established, lessons should include much work using both equations during the same lesson. These two forms should be well known before going to lesson 7 where the missing numeral will appear in many places in the equation.

Pupil Pages 47and 48: Direct the children to complete these equations. If they need to use set materials to do this, they should be allowed to do so.

$4 + 1 = \boxed{5}$ $2 + 3 = \boxed{5}$

$3 + 0 = \boxed{3}$ $0 + 1 = \boxed{1}$

$2 + 2 = \boxed{4}$ $1 + 3 = \boxed{4}$

$1 + 5 = \boxed{6}$ $1 + 1 = \boxed{2}$

$5 - 1 = \boxed{4}$ $3 - 2 = \boxed{1}$

$1 - 1 = \boxed{0}$ $4 - 2 = \boxed{2}$

$3 - 1 = \boxed{2}$ $2 - 1 = \boxed{1}$

$4 - 3 = \boxed{1}$ $5 - 2 = \boxed{3}$

VII-7. Finding missing numbers in equations

Objective: To introduce sum and remainder

To introduce finding missing numbers in equations, other than sum and remainder

Vocabulary: Sum, remainder

Materials: Flannel board, flannel board materials, chalkboard

Suggested Procedure:

In addition equations the children will have used a box to represent the missing sum. Present the equation $2 + 1 = \square$. Discuss with the children that in this equation there are a pair of numbers which, when added, give a *sum*.

Present the equation $3 - 1 = \square$. Discuss with the children that in this equation there are a pair of numbers which, when the second is subtracted from the first, give a result known as the *remainder*.

Present other equations such as:

$4 + 1 = \square$

$5 - 2 = \square$

$2 + 2 = \square$

$3 - 0 = \square$

$4 - 2 = \square$

$1 + 3 = \square$

Discuss in each case whether the sum or remainder is missing.

Pupil Page 49: Direct the children to put red x's in all boxes in which the numeral is a *sum*. Put blue x's in all boxes in which the numeral is a *remainder*.

Tell the children that sometimes the equation is written in such a way that the sum is known but one of the pair of numbers to be added is missing.

Present problems such as:

$4 + \square = 4$
$1 + \square = 4$

$2 + \square = 5$
$3 + \square = 4$

$1 + \square = 3$
$1 + \square = 5$

In these equations we see that the sum is known but that one of the pair of numbers to be added is missing. The number which must be added to the first number is missing.

The following problem would serve to illustrate the need for this kind of equation: $2 + \square = 5$.

> Two boys are playing a game of hide and seek. Some more boys came to play and now there are five boys playing together. How many boys joined the ones who were playing at first?

The children could check their answer by actually having two boys stand and see how many more had to join them so there were 5 in all.

Children can also use flannel board materials to solve this type of equation: $4 + \square = 5$.

Place 4 counters on the flannel board. Ask a child to join a set so there will be 5 counters in all.

Next consider an equation of the following form: $3 - \square = 1$.

> What numeral are we trying to find when the box is in this place in the equation? (What number is subtracted from 3 if the remainder is 1 ?)

The problem below would illustrate when this equation would be appropriate.

> Billy had three balloons. Some of them broke so he has only one left. How many have broken?

To help find the answer the children can use set material or flannel board materials. Ask a child to place a set with 3 members on the flannel board. Ask a child to remove a subset so that the remainder set has one member. Children will see the relationship between subtraction and addition when they realize that the number of members in this subset is also the number which should be added to 1 to get 3.

Pupil Pages: No pupil pages have been provided for giving children individual practice. Should you wish to provide such, do so.

$3 + 2 \underset{\text{(red)}}{=} \boxed{5}$

$5 - 3 \underset{\text{(blue)}}{=} \boxed{2}$

$4 + 1 \underset{\text{(r.)}}{=} \boxed{5}$

$5 + 0 \underset{\text{(r.)}}{=} \boxed{5}$

$3 - 2 \underset{\text{(b.)}}{=} \boxed{1}$

$4 - 1 \underset{\text{(b.)}}{=} \boxed{3}$

$2 + 2 \underset{\text{(r.)}}{=} \boxed{4}$

$2 - 1 \underset{\text{(b.)}}{=} \boxed{1}$

$3 - 1 \underset{\text{(b.)}}{=} \boxed{2}$

$2 - 2 \underset{\text{(b.)}}{=} \boxed{0}$

$1 + 2 \underset{\text{(r.)}}{=} \boxed{3}$

$3 + 0 \underset{\text{(r.)}}{=} \boxed{3}$

$5 - 4 \underset{\text{(b.)}}{=} \boxed{1}$

$5 - 2 \underset{\text{(b.)}}{=} \boxed{3}$

$1 + 1 \underset{\text{(r.)}}{=} \boxed{2}$

$2 + 3 \underset{\text{(r.)}}{=} \boxed{5}$

$4 - 3 \underset{\text{(b.)}}{=} \boxed{1}$

$5 - 1 \underset{\text{(b.)}}{=} \boxed{4}$

Chapter VIII

PLACE VALUE AND NUMERATION

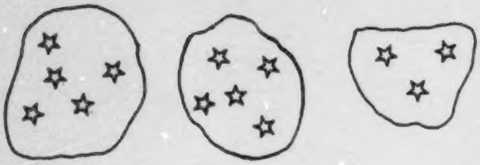
Background

The fundamental problem which we solve in this chapter is that of assigning names to numbers greater than nine. We have named the first few numbers: 0,1,2,3,4,5,6,7,8, and 9, but the procedure of assigning a new name to each successive number is clearly impractical. Some sort of system of naming numbers is necessary. This chapter is devoted to the Hindu-Arabic system of numeration which we use. It is interesting to notice that this is a relatively modern system--quite unknown to the Greeks and Romans. Indeed, mathematicians have conjectured that the rather feeble accomplishment of the Greeks in algebra was probably due to their lack of a reasonable notational system. The system which we now use is only about a thousand years old: it was carried to Europe, along with spices and sandalwood, by Arab traders.

The simplest numeration systems are very closely related to tallying. For instance the Romans used I, II, III, and IIII for the first four numbers. Of course this sort of notation is completely impractical for large sets, and people soon found ways of simplifying the naming system. The first step was to count by groups of some agreed upon size, so that, for example, we might refer to seven dozen eggs, or a gross of pencils.

Let us state in mathematical terminology just what this sort of "grouping" amounts to. Suppose we are trying to describe a set which has a great many members. We select a subset of some standard number of members (like a dozen, or a gross) and partition (split up) the set into as many equivalent subsets as possible. There may or may not be a remainder set (that is, a subset left over). Thus, if 5 is the standard number, we may partition the set

into the sets



and describe the original set as consisting of 2 fives and 3 ones. The number of members of the standard subset is more or less arbitrary. Thus we may describe the number of members of the set pictured above in any of the following ways:

fives	ones
2	3

threes	ones
4	1

tens	ones
1	3

We customarily group by tens--presumably because we have ten fingers. IBM machines customarily group by twos, and the barefoot Mayans grouped by twenties.

This system of counting by groups has been used by most civilizations. But as greater and greater numbers of objects were considered, new names for greater and greater standard numbers became necessary. Thus the Romans used I, V, L, C, D, and M for one, five, fifty, one hundred, five hundred, and one thousand. At each stage, as names for greater numbers were needed, a new symbol was needed. But the Hindu system circumvents this difficulty by assigning meaning to the place a digit occupies, and manages to create numerals for every number from the ten symbols: 0,1,2,3, 4,5,6,7,8, and 9. This is a truly remarkable achievement.

The idea of grouping, together with place value, is enough to permit us to assign numerals to the first hundred numbers. The step from the pattern: to the numeral: 47 is a simple one, and it should be clear that this number is to be assigned to a set which consists of 4 tens and 7 ones. The number 10 is described in precisely the same way: this is the number which is to be assigned to a set of 1 ten and 0 ones. We say that the right hand digit is in the ones' place, and that its neighbor on the left is in the tens' place.

tens	ones
4	7

There is a further step in our system of numeration. Suppose that a set consists of 23 tens and 4 ones. In counting the 23 tens we would normally group these in tens, so that our record keeping might look like either of the following:

tens	ones
23	4

tens of tens	tens	ones
2	3	4

In either case, naming this number as 234 is completely natural. We say that the right hand digit is in the ones' place, its left hand neighbor in the tens' place, and the next left hand digit is in the hundreds' place. We usually call tens of tens "hundreds," and tens of tens of tens we call "thousands." But the practice of naming these greater numbers eventually becomes impractical and we fall back on the numerals. Thus

234,468,789,345,863,456,998,567,452,345,765,989

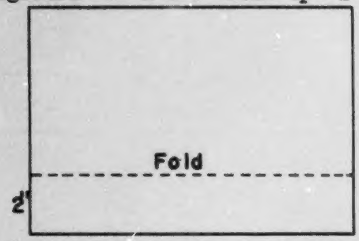
names a certain number in a perfectly well defined way, but it is doubtful if many of us remember the ordinary names beyond quadrillion.

In order to be sure that you understand our numeration system you might want to imagine the following whimsical situation:

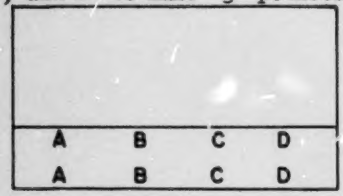
> On Mars there are several forms of life, and the dominant form is called the dozer because of its habit of taking cat naps. The dozer has neither hands nor feet, but it manages manipulations very nicely using its twelve tentacles. The dozers are mathematically accomplished (all little dozers doze through calculus in the first grade) and they have invented a system of numeration which is quite similar to ours. But of course they count by dozens, for anatomical reasons. They use the numerals 0,1,2,3,4,5,6,7,8, and 9 just as we do, but they use N for ten and L for eleven. It is curious that they have not found it necessary to invent a digit for twelve (or is it?). Naturally the counting is done by dozens and every little dozer understands that N7 is ten dozens and 7 ones, whereas 7N is 7 dozens and ten ones. They also count by dozens of dozens (which they call gross), so that L5N means eleven dozens of dozens, 5 dozens, and N ones. Now the problems:
>
> (a) How do the dozers write twelve? (b) What would we call the number which they label 100? (c) 7 + L = ? (of course the answer must be written so that dozers can read it)
> (d) A little dozer undulated down to the store with 2N shekels to buy L shekels worth of licorice. How much change did he bring his parents?

HOW TO MAKE "SHOW-ME" CARDS

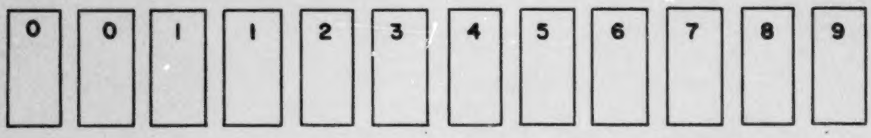
1. Use a piece of tagboard 6" x 6". Fold up 2" from the bottom.



2. Staple at A, B, C, and D to make 3 pockets, each almost 2" wide.



3. Cut a strip of tagboard 18" x 4" into 12 strips $1\frac{1}{2}$" x 4". With felt pen, write numerals as follows:



4. Children should be taught early to lay out numeral cards in order on their desks and to replace them in order.

5. In the game, pupils figure out the solution to a problem the teacher gives orally or on the chalkboard. Children place the numerals for the answer in the pockets, hold the cards against their chests with the answers concealed until the teacher says, "Show-me!" Then all turn cards and hold them, answers toward teacher, against their chests while teacher makes a quick survey to see who was right.

VIII-1. Partitioning sets into equivalent subsets, with and without remainders

Objective: To help children understand that a set may be partitioned (separated) into equivalent subsets and that there may or may not be a remainder set

Vocabulary: Review words "partitioning," "equivalent," and "remainder set" in new context

Materials: Children's set materials

Suggested Procedure:

Children use own set material to count out set with 8 members. Direct them to make 2 more sets that are equivalent to the first.

How many sets with 8 members do we have now? (3)

Have children put the members of the 3 sets into one set.

Now count 4 members of your set and pull them just a little to one side.

What do we call this part of the set? (subset)

Have them keep on counting out subsets with 4 members each until they have made as many subsets as they can.

How many subsets did you have when you finished? (6)
Did each subset have 4 members? (Yes)
Could we say we have 6 fours in our original set? (Yes)
We have partitioned the original set into 6 sets with 4 members each.

Have children put all subsets into one set again and then partition into subsets with 6 members each, using above procedure. When 4 sets of 6 have been obtained and discussed, ask children to partition the set of 24 members into sets of 5 members each.

> How many subsets do you have? (5)
>
> Does each subset have 5 members? (No, all but one.)
>
> How many subsets have 5 members? (4)

Direct children to look at the set that does not have 5 members. Call it the remainder set, the set that remains when all the equivalent subsets have been made.

> How many members does the remainder set have? (4)

Go to one child's desk and hold up one member of his remainder set. Ask if this is one member. Keep it in your hand as you hold up another member and ask if this is one member. Ask the same question about each of the 4 members.

> If each of these is one member, could we say this subset has 4 ones? (Yes)
>
> Can we say we have 4 subsets of 5 members and a remainder set with 4 members or 4 ones? (Yes)

Use the same procedure and partition the set material into subsets with 3 members, then 7 members, each time discussing how many equivalent subsets there are and whether or not there is a remainder subset of some ones.

> Can anyone figure out why we sometimes have a remainder set and sometimes do not?
>
> Instead of saying that there is no remainder set, could we say that the remainder set is empty, or that it has 0 members? (Yes)

Pupil Pages 50 and 51: Circle sets of 5 (etc.), and write numerals showing how many sets and how many ones in remainder set.

Further Activities:

1. Have children choose 12 objects from set boxes and let individual children suggest ways of partitioning. ("Bill, how many things shall we use for our first subset?") Make sure that sometime during the period it is noticed that partitioning into subsets of 2, 3, 4, and 6 will not yield a remainder set, while some other partitions will.

   What would happen if we used only one member in the first subset?

   What would happen if we used 12 members in our first subset?

2. Have children partition 36 objects into equivalent subsets with various numbers of members. Keep a record on chalkboard of the results.

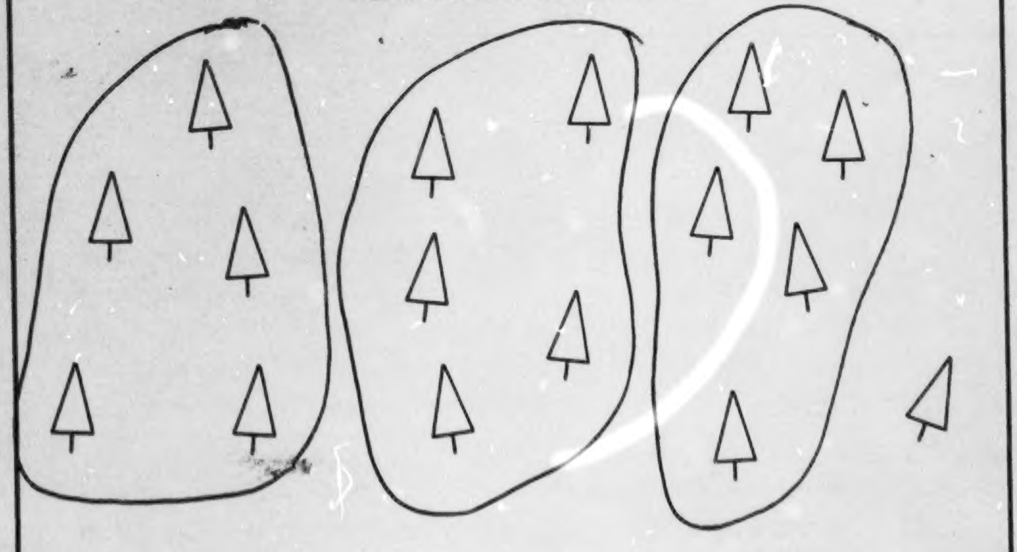
   4 sets of 9 remainder set 0 ones
   4 sets of 8 remainder set 4 ones
   5 sets of 7 remainder set 1 ones
   6 sets of 6 remainder set 0 ones

   Be careful not to use partitions above 9.

3. Put sets of more than 20 objects on flannel board (circles, stars, etc.). Have children use pieces of yarn to partition into subsets of various sizes, with or without remainder sets.

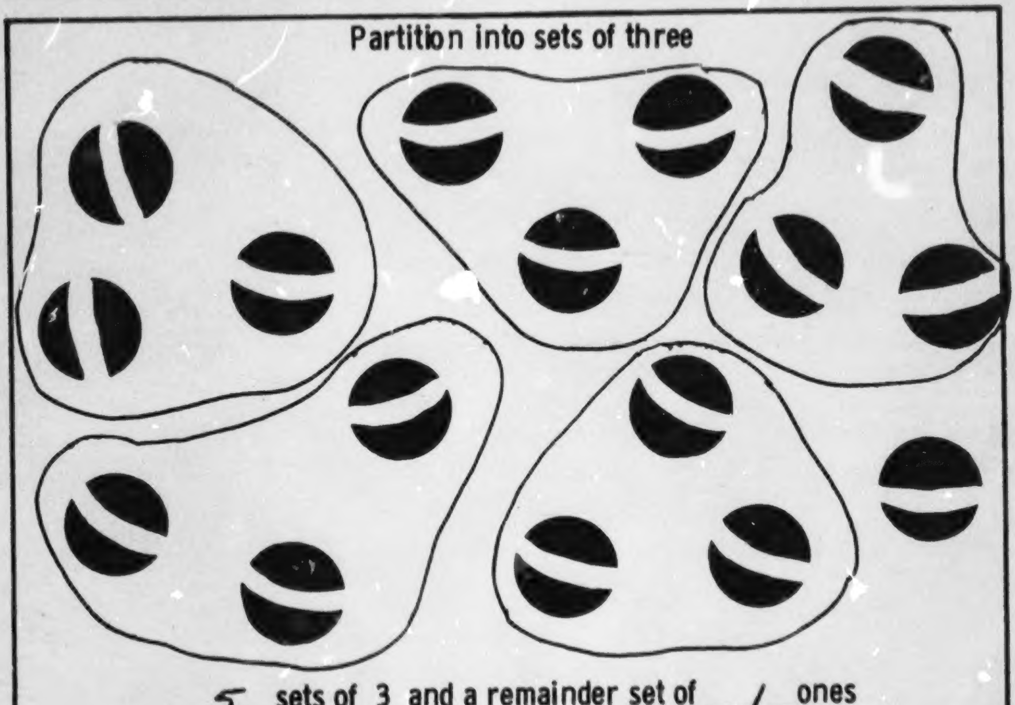
4. Draw sets on chalkboard (balls, trees, kites, etc.) and ask children to draw rings around equivalent subsets of various numbers of members. The remainder set (unringed) should be discussed in terms of so many sets of ? and a remainder set of ? ones.

Partition into sets of five



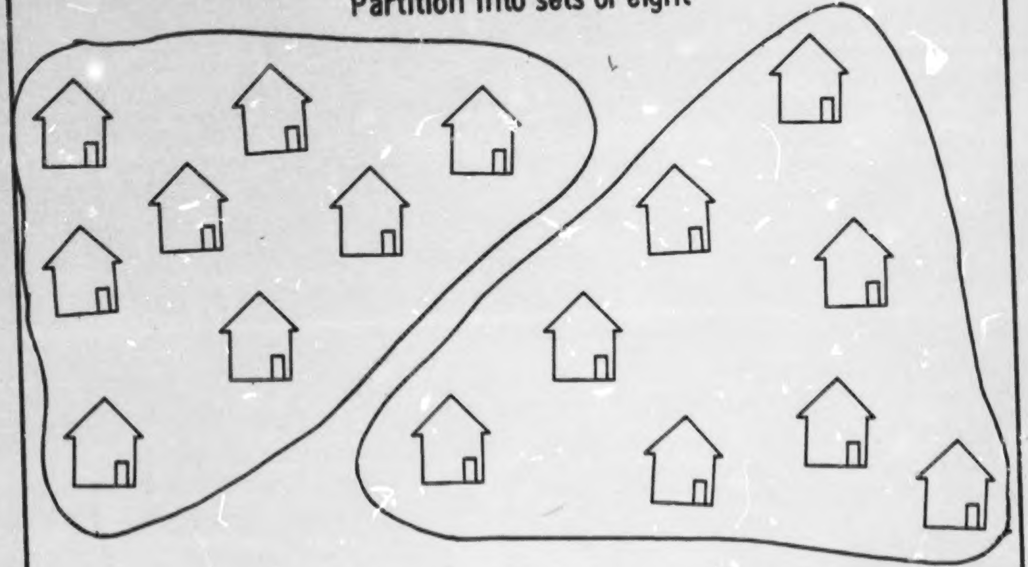
3 sets of 5 and a remainder set of 1 ones

Partition into sets of three



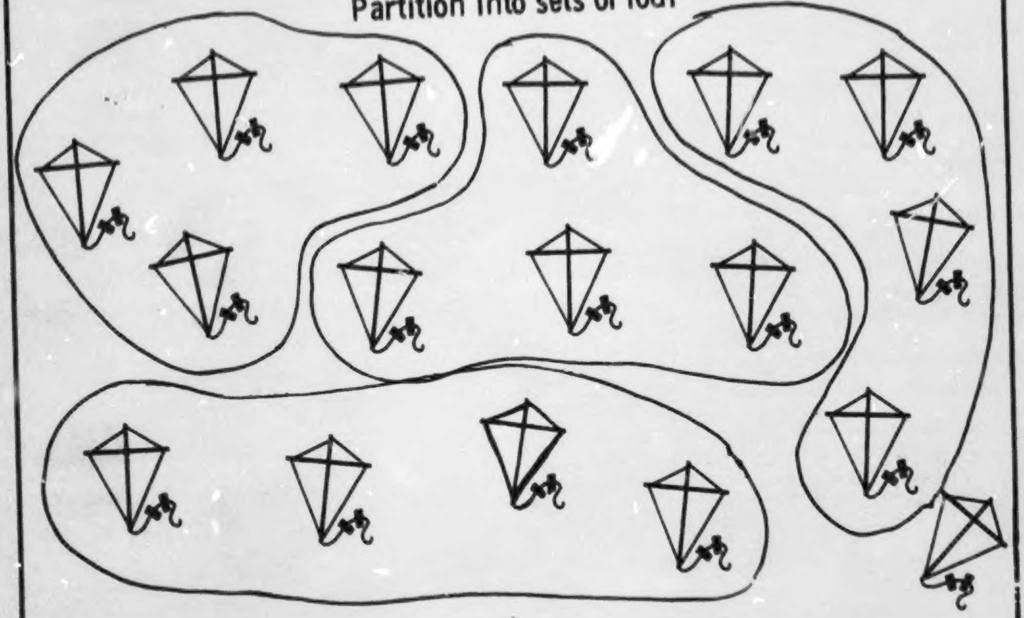
5 sets of 3 and a remainder set of 1 ones

Partition into sets of eight



__2__ sets of 8 and a remainder set of __0__ ones

Partition into sets of four



__4__ sets of 4 and a remainder set of __1__ ones

VIII-2. Counting by sets of ten

Objective: To help children learn to count sets of ten

Vocabulary: Ten (printed word), tens

Materials: Several kinds of counting material for tens, some large enough for teacher to use in front of class, some small enough for use by each pupil. Each set of material should contain an even multiple of ten, from 50 to 90. Count out the multiples of 10 before you start. Suggested materials: blocks, beads, buttons, spools, popsicle sticks, theater tickets, beans, pegs, paper clips, 9 strings or shoe laces for stringing, or dowels for objects with large holes.

Suggested Procedure:

Ask a child to count ten spools or beads for stringing on a string. (Do not tie any other object on the string to hold the spools or beads on.)

Use another piece of string, and ask another child to count the objects for stringing. Continue stringing ten on each string until you have 9 strings altogether.

> How many spools did I string on each of these strings? (10)

Hold up 3 strings.

> How many sets of 10 spools do you see here? (3)
> Can we call this 3 tens? (Yes)

Ask further questions about the tens of spools, then use similar procedure with various other types of material: piles of 10 blocks, bundles of 10 popsicle sticks with rubber bands around them, etc. Emphasize the fact that the number of tens has nothing to do with the *kind* of material counted.

Give each child a set of the materials to count, and have each count the objects into sets of 10. (If your children work well in small groups, you may prefer to have several work together.) Have each child (or each group) report how many tens there were in the set counted. Make a list on the board, as follows:

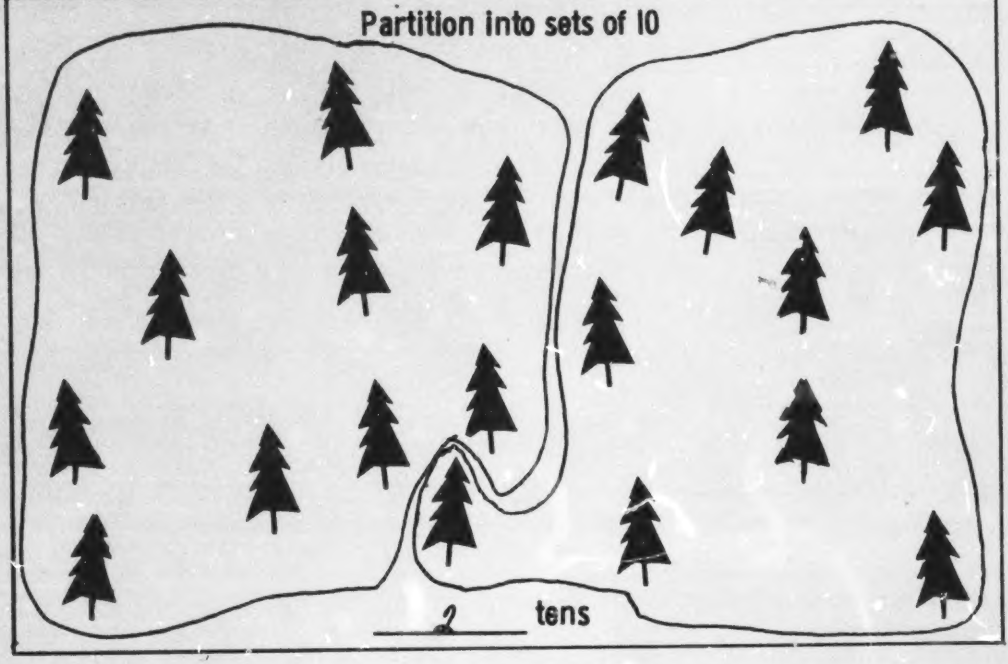
tens
3
6
etc.

Pupil Pages 52 and 53: Circle sets of 10, and write the numeral for the number of sets in the blank.

Further Activities:

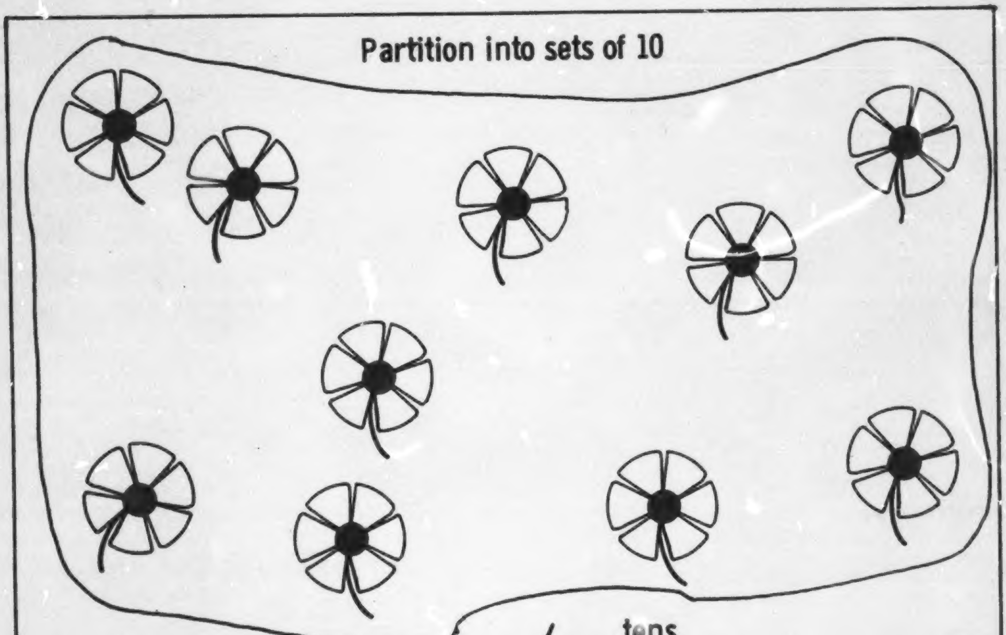
1. Have children count sounds as you tap the desk or a triangle. After a set of 10 taps, child holds up a finger. Stop on every multiple of 10, and have children tell you how many tens of taps there were.

Partition into sets of 10



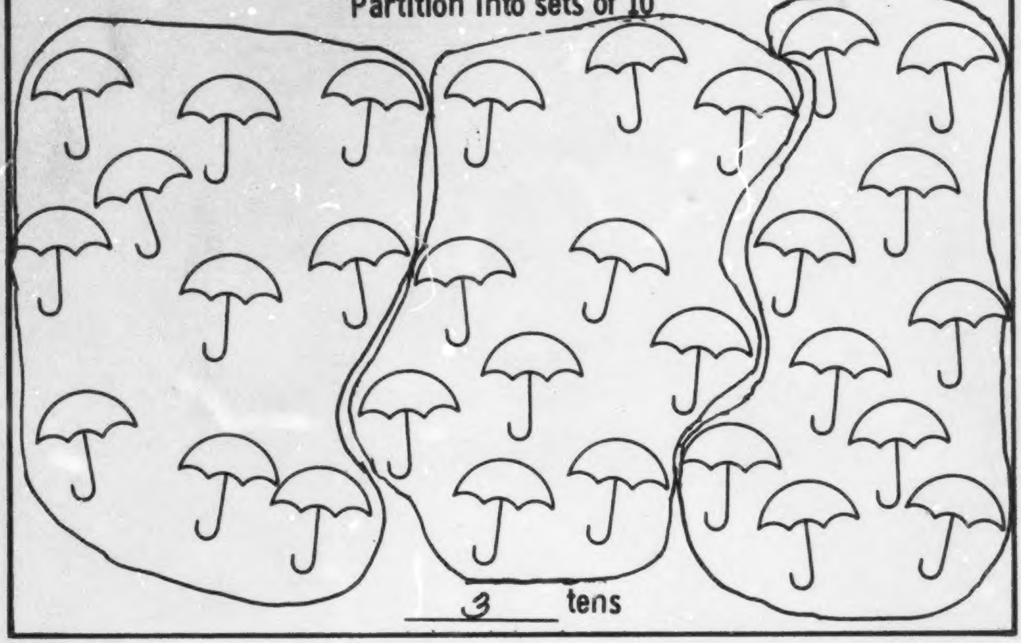
2 tens

Partition into sets of 10



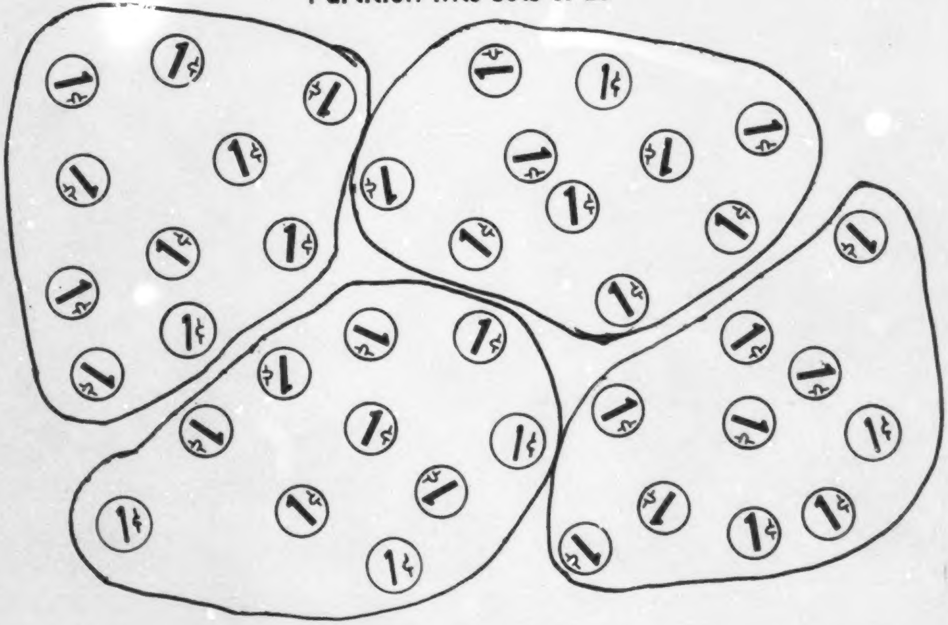
1 tens

Partition into sets of 10



3 tens

Partition into sets of 10



4 tens

VIII-3. More on the number ten

Objective: To help children understand ten as ten ones or one ten

Vocabulary: No new terms

Materials: Blocks, sticks, flannel board circles, paper clips, theater tickets in sets of 10 of each

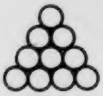
Suggested Procedure:

Place ten circles on the flannel board in a random arrangement. Remove one circle; hold it up.

How many things am I holding? (One)

Replace that circle. Pick up another, and repeat question. Hold up a single stick, block, ticket, etc., and ask how many you are holding.

Rearrange circles on flannel board into triangle arrangement:



How many things are on the flannel board? (10)

Have children count sticks as you bundle them together. Ask how many are in the bundle and put a rubber band around it.

Count blocks and make a "pillar" of 10.

How many sets of ten blocks are there? (One)
How many ones are in the stack? (Ten)

Pull out one stick from the bundle and ask how many. Put it back and ask how many *ones* are in the bundle. Ask how many *tens* are in the bundle.

Further Activities:

If this concept seems difficult, you may want to use theater tickets: A strip of 10, torn apart into 10 ones, can easily be put back together with masking tape as a strip of 10 again. Paper clips, also, may be strung together to show that ten ones and one ten are the same number.

Chalkboard:

Draw a set of at least 40 objects. Have children draw rings around sets of ten.

NOTES

VIII-4. Counting by tens and ones

Objective: To help children learn to count sets of ten with a remainder set

Vocabulary: No new terms

Materials: Flannel board squares and strips for ten, similar flannel board material, other types of counting material

Suggested Procedure:

Place one set of ten on flannel board. Have children count to determine how many. Be sure they understand that the name that tells how many is "ten" or "ten ones." Have a child place a matching set without counting. Have pupils note that there are now two tens. Do the same for a third set of ten. Then place 4 ones.

> How many sets of ten do we have? (3)
>
> What number tells how many are in the set that is not a ten? (4)
>
> How many ones are in this set? (4)
>
> How can we tell how many are in the whole set? (3 tens and 4 ones)

Put 7 more objects on the flannel board to make four sets of 10.

> Now how many sets of ten do we have? (4)
>
> If we have partitioned all our material into sets of ten, what can we say about the remainder set, the set of ones left over? (the empty set)
>
> What number do we use for the empty set? (0)
>
> Then how many sets of ten do we have, and how many ones? (4 tens and 0 ones)

Use other types of material to develop understanding of partitioning into tens and ones.

Let children use material (sticks, pegs, bottle caps, theater tickets, etc.) to partition into tens and ones. Keep a record on the chalkboard of their results in the following form:

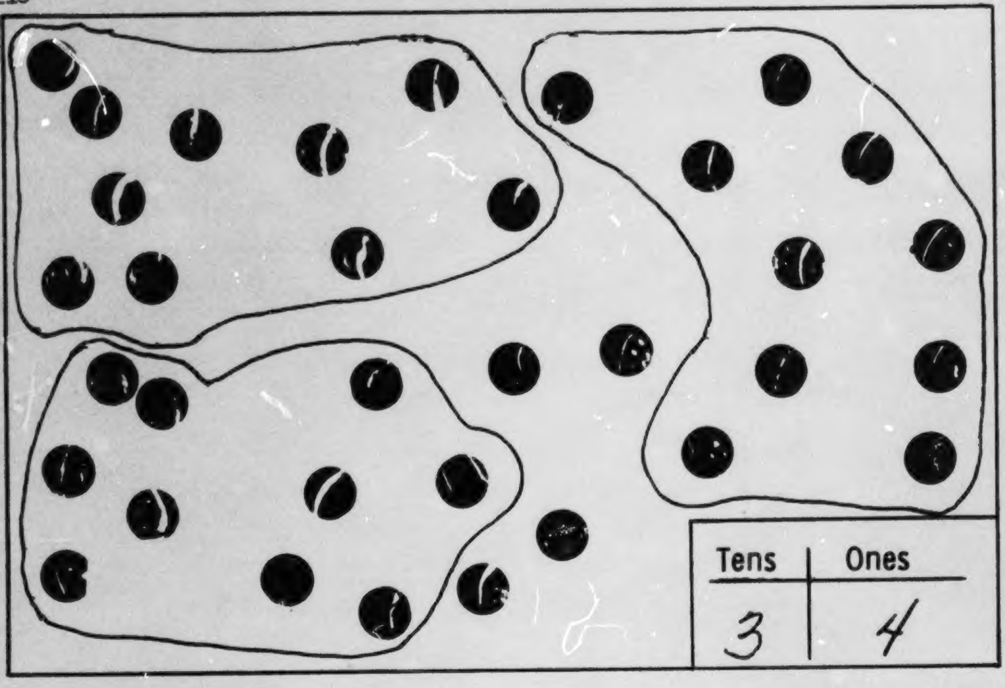
	tens	ones
Tom	3	5
Dick	6	1
Harry	2	8

Ask which column shows larger sets. (tens column) Discuss which child must have had most objects (Dick, with 6 tens and 1 ones or Harry with 2 tens and 8 ones.) Have children show their materials to class if there is a question. (Discourage counting by ones up to the total amount.)

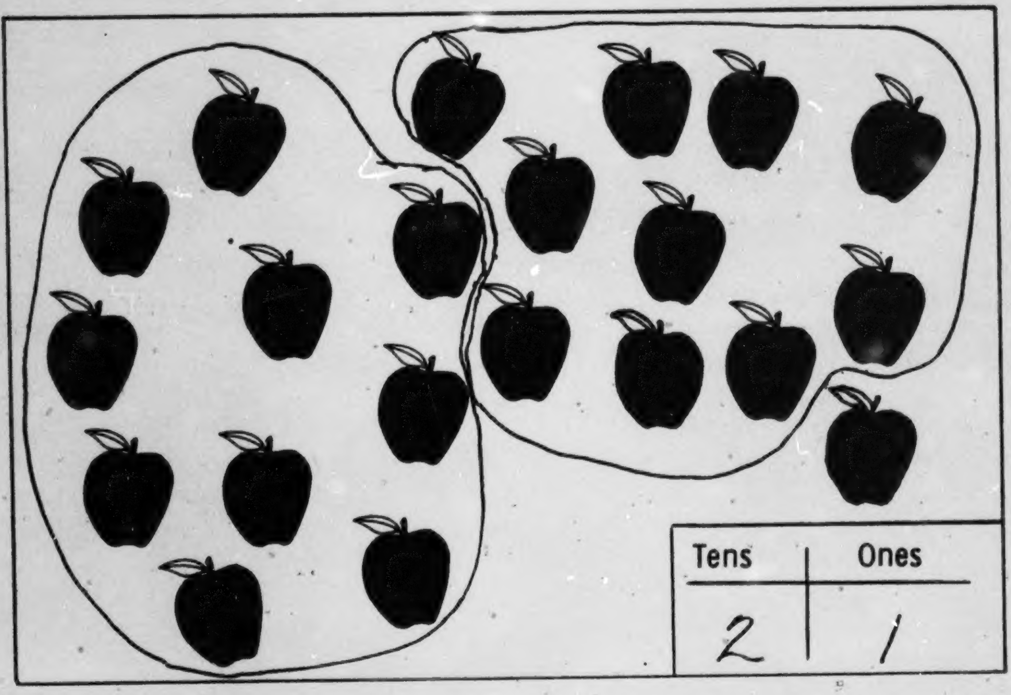
Pupil Page 54: Draw rings around sets of 10, and enter numerals for numbers of tens and ones at bottom.

Further Activities:

Have children counted. Put one set of ten in one corner, another in another, etc. Let the "ones" remain at their seats or stand in the middle of the room. Find how many tens and ones there are. Try it with boys and girls separately. Let the "ones" trade places with pupils in the "tens sets" so that they see that a ten is simply a set of ones.



Tens	Ones
3	4



Tens	Ones
2	1

VIII-5. Spoken names for multiples of ten

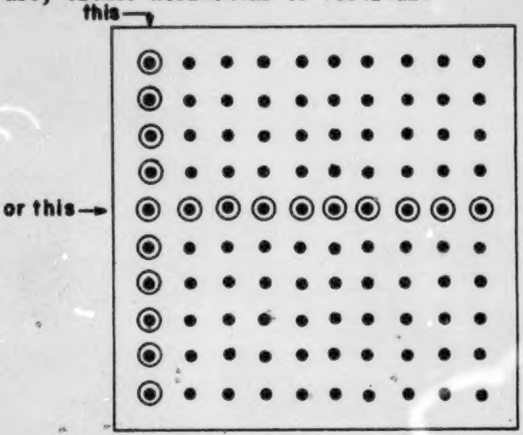
Objective: To teach the names for multiples of ten

Vocabulary: Ten, twenty, thirty, ... ninety

Materials: Flannel board material, small squares for ones strips of ten small squares; a large piece of plain tagboard or cardboard; spool board with spools or peg board with pegs. (See suggestions at beginning of unit)

Suggested Procedure:

Direct a child to come to the spool board and place ten spools on any row he wishes to use, either horizontal or vertical.



Discuss how many spools it took to fill the row.

Indicate a row parallel to the one with the spools.

How many spools will this row hold? (10)

How do you know it will hold 10 spools? (Rows of pegs match so there will be the same number of spools in each row.)

Place 3 rows of spools parallel to the first row.

> How many sets of 10 spools do we have now? (4)

Place on the flannel board a set of 10 squares. Ask a child to tell how many there are. Ask another child to place a square under each square on the flannel board.

> How many sets of 10 pieces do we have now? (2)

Leave a space under the rows already on the board, and arrange 7 more strips of 10 under the 2 rows already there. (Sweep your hand under each row from left to right when counting.)

> Does anyone know how many are in the first two rows altogether? (Two tens is, of course, a correct answer. However, here we are seeking another name for 2 tens--i.e., "twenty."

Point out that you can count "ten, twenty ..." and ask whether anyone knows a word for 3 tens.

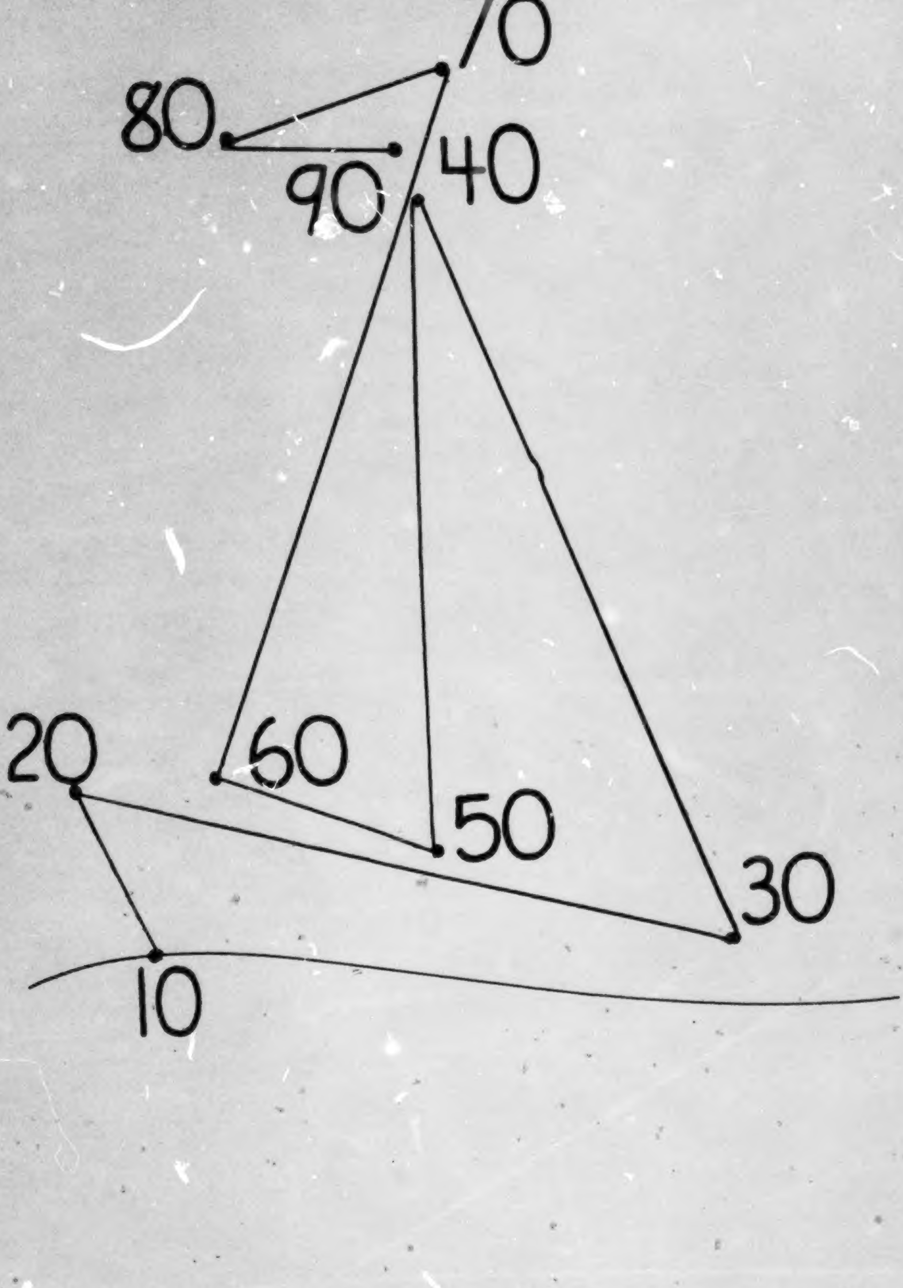
Some children will probably know how to count by tens. If not, count for them, sweeping your hand across under each row as you count, "ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety."

Use the sheet of tagboard and cover all of the tens except the 3 rows at the top. Ask the word that tells how many when we have 3 rows with 10 members in each. Continue moving the tagboard down, one row at a time. Ask the same question each time. Continue using the tagboard procedure but skip around and use a variety of tens. Ask the children to describe how many sets with ten members each and to tell the name that can be associated with the whole set. Be sure the children say the word properly (thirty, not thirteen. It will be helpful, if they have progressed far enough in reading, to write the words on the chalkboard and point out the "ty" ending.)

> If a set has twenty members in it, how many subsets of ten has it? (2)
>
> What word tells us the number of members in 4 sets of ten? (40)

Pupil **Page** 55: Draw line segments in order between multiples of 10.

NOTES


70
80
90
40
20
60
50
30
10

VIII-6. The spoken names of the numbers from 21 through 99

Objective: To help children understand how to count sets of more than twenty members using the spoken names for the numbers

Vocabulary: The spoken names of numbers from 21 through 99

Materials: Different kinds of counting materials in groups of 20 or more. Some sets should provide remainder sets.

Suggested Procedure:

If your children work well in groups, they may be arranged around tables or desks pulled together. Then each group would be given a set of material to count. Otherwise, give each child a set of material.

Direct the children to count the set material into subsets of ten, to make as many sets with ten members as possible, and then to be ready to report to the class how many tens and how many ones they had.

While the groups are counting, place 2 tens and 5 ones on the flannel board.

> Who can tell about the set of squares on the flannel board? (Two tens and 5 ones, or possibly 25)
>
> If you want to count the squares, how can you do it? (Probably, the children will begin "1, 2, 3, etc.")

Cover the 5 ones with tagboard.

> Can we count the 2 tens a shorter way? (Ten, twenty)
>
> Then let's go on from twenty: twenty-one, twenty-two, twenty-three, twenty-four, twenty-five.

Ask several groups or individuals to report how many things they had counted by reporting first tens and ones, and then by counting multiples of ten and going on from the last multiple with ones. As they give tens and ones, enter their results on a chart on the board. When they have told how many altogether, point to the chart and say, "or 6 tens and 5 ones."

Pupil Page 56: Count the tens and ones and put the answer on the chart in each box.

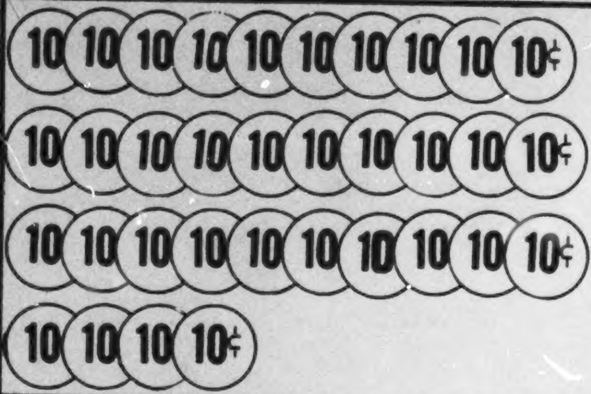
Further Activities:

Different kinds of counting material may be needed for some children. Others may need to verify the possibility of counting first by tens and then by ones, by counting the whole set by ones to see that they arrive at the same number.

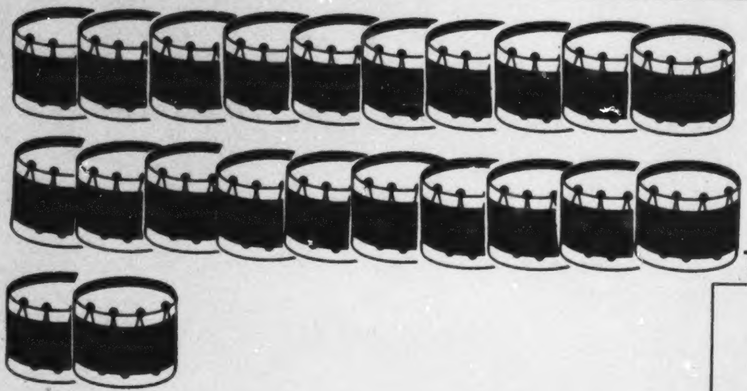
1. Let children count taps on desk or triangle, counting to themselves to ten, then saying "ten" and holding up a finger, continuing to twenty and holding up another finger, etc.
2. Have children construct a set involving sets of tens and ones. Using tabulation form

	tens	ones
Picture of peg	2	6
Picture of stick	4	9
Picture of bottlecap	3	8

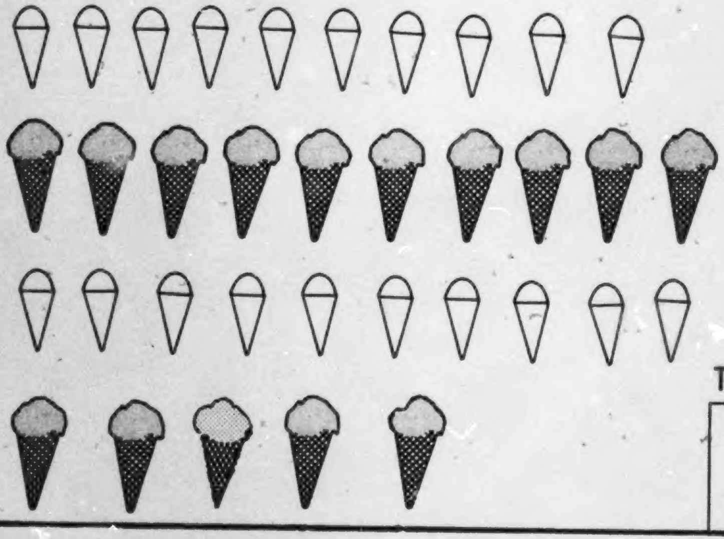
assign children to make the set indicated in the table either with flannel board materials or with sticks, pegs, bottle caps, etc.



Tens	Ones
3	4



Tens	Ones
2	2



Tens	Ones
3	5

VIII-7. The written numerals 20 through 99

Objective: To help children associate the correct written numerals, as well as spoken names, with the numbers 20 to 99

Vocabulary: No new words

Materials: Different kinds of set materials: blocks, sticks, flannel board squares, pegs, etc. ("SHOW-ME" cards for further activities)

Suggested Procedure:

(This activity should follow many experiences with counting and naming sets of ten with and without remainders and with tabulation in chart form as used in lesson 4.)

Place 3 "pillars" of 10 blocks each on chalk tray and have children tell in two ways how many there are. (3 tens, thirty)

Place 5 sets of 10 on flannel board. Again have children tell in two ways how many there are. Make 2 bundles of 10 sticks each and put them with 6 sticks on a table. Have this set named. (2 tens and 6 ones, twenty-six)

On the flannel board, show 4 sets of ten and 7 ones.

Using tabulation form as in lesson 4, have children tell where you will write the numeral for the tens and the numeral for the ones for each group of objects. Then to the right of the tabulation, write the numeral for the set.

tens	ones	
3	0	30
5	0	50
2	6	26
4	7	47

Show children that the digits are written close together and without the line between tens and ones, but that the digit on the right means ones and the digit to the left of it means tens.

> **NOTICE**: It is very important that this way of speaking be used rather than saying that the digit on the left means tens. Considerable later confusion with hundreds and thousands can be avoided if the emphasis is on: "The ones on the right and the tens to the left of them."

Pause frequently to ask about numerals:

> In 42, what does this (point to digit 4) mean? (4 tens)
> What does the 2 mean? (2 ones)

Continue with teacher-written notation until you feel the ideas are clear. AVOID THE -TEEN NUMBERS. Stay above 19.

Have individual children count sets of materials in tens and ones. Let them write the numeral on the chalkboard and read it. (Count "two tens and one one," write "21," read "twenty-one.") Be sure many opportunities are given to write zero ones and to read exact multiples of 10.

Pupil Page 57: Have children write numeral opposite tens and ones tabulation, or tens and ones opposite numeral.

Further Activities:

When children seem ready to work without manipulative objects, write numerals on chalkboard and have them draw a ring around the digit that indicates the number of tens or the number of ones.

Give each child a "SHOW-ME" card. Ask children to show the numeral for 3 tens and 7 ones, etc., or for thirty-seven, twenty-eight, etc.

Let children work in groups, if possible. Have on chalk tray numeral cards for numbers between 20 and 99. Let a member of each group choose a numeral, then the group will count out material in tens and ones to make the set named. Unused (unnecessary) material should be replaced before another group checks to see whether the first is correct.

Draw 10 rows of 10 objects (X's perhaps) on the chalkboard. Point to or draw a ring around one of them and have children count by tens (ten, twenty, thirty, ...) and ones to find out how many objects up to and including the object identified.

NOTES

Tens	Ones
2	3

is 23

47 is

Tens	Ones
4	7

Tens	Ones
3	0

is 30

Tens	Ones
7	2

is 72

Tens	Ones
5	5

is 55

87 is

Tens	Ones
8	7

Tens	Ones
9	6

is 96

VIII-8. Spoken names and written numerals for the numbers 10 through 19

Objective: To present spoken names and written numerals for the numbers 11 through 19

Vocabulary: Eleven, twelve, thirteen, ..., nineteen

Materials: Flannel board materials, blocks, sticks, theater tickets, etc. (one ten and ten ones of each)

Suggested Procedure:

Place ten objects on the flannel board and have children name the number two ways, as one ten or ten ones.

Underneath, at the left, place another object.

> How many do we have now? (one ten and one one, or one more.)

Ask children to go back to the beginning and count by ones. If they hesitate, supply the word "eleven."

Using the same procedure, place another object and develop the idea of twelve as 10 and 2 more, or twelve ones. Continue with thirteen. Be sure children know they are saying thirteen, not thirty. It would be good with some groups to write the word thirteen on the chalkboard. (Notice that if the names for sets with one ten and some ones followed the same pattern as the other number names pupils have been studying, we would say something like "onety-one, onety-two, etc." It would not be quite so hard if we said, "Teen-one, teen-two, teen-three, teen-four," but that just isn't the way it's done in the English language!) Use other sets of materials and emphasize the oral names, and the idea of one ten and so many ones. Use the tabulation form, then the written numeral to connect them. With most groups, it will probably be wise to stop at "thirteen" the first day. Take the rest of the teens another day (or two!). A thorough understanding of the teens will save much difficulty later on.

Pupil Page 58: Have children write numeral opposite tens and ones tabulation, or write tens and ones opposite numeral.

Further Activities:

Give many opportunities for counting sets fewer than 20 into tens and ones, recording and saying the result as tens and ones and as the -teen number.

1. Have children build sets from directions "So many tens and so many ones," and "Show me a set of fifteen," or "Make a set of 12" (using written numeral). Be sure to use "one ten and zero ones for 10."
2. Write numerals on chalkboard and have children draw a ring around the part that tells tens or the part that tells ones.
3. With "SHOW-ME" cards have children show the numeral from ten-and ones-instructions and from spoken word.
4. When you are sure children have oral and written names for -teen numbers firmly established, reintroduce numbers greater than twenty, and check to make sure they do not confuse 12 with 21, etc.

21 is

Tens	Ones
2	1

15 is

Tens	Ones
1	5

Tens	Ones
1	2

is 12

Tens	Ones
5	0

is 50

40 is

Tens	Ones
4	0

14 is

Tens	Ones
1	4

VIII-9. Inequality for numbers 0 through 99

Objective: To establish the idea of "greater than" and "less than" for numbers 0 to 99

Vocabulary: No new words; review greater than, less than

Materials: One container of 28 sticks or other small counting material (theater tickets) capable of being bundled into easily recognizable tens, and one container of 31 of the same kind of material

Suggested Procedure:

Empty the two containers on a table or desk. Ask the children which set of material has more members. Then have a child determine which has more members by using one-to-one correspondence. Point out that this is a good way to find out which set has more members, but that if we wanted to find out which was greater, the number 89 or the number 91, it would take a long time to count out sets of those sizes and use one-to-one correspondence to find out which has more members.

Write the numerals 16 and 6 on the chalkboard and ask a child to tell which numeral names the greater number. Ask him how he can tell.

> Which is greater, a number with 1 ten or a number with 0 tens?
>
> Is 10 greater than 9 ? Is 10 greater than 8 ?
>
> Which is greater, a number with 2 tens or a number with 1 ten? Which is greater, 25 or 17 ? Etc.

Use original containers of material again. Have a child count each of the sets into tens and ones. Bundle the tens so that you can show them, "This ten corresponds to that ten; this ten corresponds to that ten. But here (in the set of 31) you have one more ten than in this set."

If we have numbers between 10 and 99, what part of the numerals should we think about first to find out which is the greater number? (tens part)

Write numerals on chalkboard and have children draw a ring around the numeral for the greater number. Have them tell you, if they hesitate, what the parts of the numerals show.

Pupil Page 59 : Top--Draw a ring around the numeral for the greater of the two numbers in each set. Bottom--Draw a ring around the numeral for the greatest number in each set.

Pupil Page 60: Write the numerals in each box in order.

Further Activities:

1. Write a numeral on the chalkboard and have children name the number that is one greater or one less than the one whose name you have written. Ask them to name the number that is ten greater or less.
2. Let children use hundreds-squared paper (10 rows of 10 squares) and write numerals from 0 to 99. This may be done occasionally for the next few weeks, at least. Numerals may be written either vertically or horizontally. To check understanding of greater and less, on chalkboard write:

12	21
45	49
62	57
38	25
70	39
98	89

Ask children to copy the pairs of numerals and draw a ring around the one in each pair that names the greater number. (They will ring 21, 49, 62, 38, etc. to provide you with an easy check of their understanding.) Another day, have them draw a ring around the numeral for the smaller number in each pair. On other days you may wish to write 3 numerals in each group and ask children to draw a red ring around the numeral for the greatest and a blue ring around the numeral for the least number, etc.

NOTES

Draw a ring around the numeral for the greater of the two numbers in each set

[✓ 36]	27	17	[✓ 24]	[✓ 30]	13
79	[✓ 81]	77	[✓ 96]	[✓ 65]	32
[✓ 21]	19	[✓ 91]	58	[✓ 28]	23
[✓ 10]	6	45	[✓ 50]	88	[✓ 95]
53	[✓ 97]	[✓ 61]	59	[✓ 56]	49

Draw a ring around the numeral for the greatest of the three numbers in each set

36	27	[✓ 40]	[✓ 11]	9	10
2	[✓ 55]	26	20	19	[✓ 21]
[✓ 31]	18	23	[✓ 46]	38	29
75	[✓ 89]	64	82	[✓ 91]	43
49	[✓ 51]	25	14	32	[✓ 73]

Write the numerals in each box in order

19 31 25 19 25 31	23 12 18 12 18 23
86 45 29 29 45 86	77 68 92 68 77 92
62 66 85 62 66 85	53 36 16 16 36 53
56 84 99 56 84 99	42 74 37 37 42 74
63 48 72 48 63 72	15 34 83 15 34 83
94 28 54 28 54 94	59 39 54 39 54 59

VIII-10. The hundreds

Objective: To develop the idea of the hundreds place in written notation

Vocabulary: Hundred(s)

Materials: Small counting materials, all of one kind--beans, corn, pegs, etc.--more than 300 of them--in a container, preferably glass or clear plastic

Suggested Procedure:

Ask children to guess how many beans are in the container. Suggest that they find out by counting, and everyone will help. Give each child a handful of beans and ask him to count the beans in sets of ten. When all have finished, put all the remainder sets (ones) together, and have them counted into sets of ten.

Ask a child to count the sets of ten. Using tabulation form, record the number of tens. Ask another child to count the ones. Record it.

Tens	Ones
32	7

How many beans do we have? (We have 32 tens and 7 ones.)

Ask them if that is the way we expected to name the number of beans.

Tell the class to help you count the tens by tens: ten, twenty, thirty, etc. When you get to one hundred, stop, and emphasize the pronunciation of the word. Print it on the board. Ask a child to count how many tens it took to make one hundred.

Make another tabulation form on the chalkboard and ask where you would put hundreds on this chart.

tens	ones
10	0

Show that the hundreds is really ten tens, and that the numeral 10 could be written in the tens column, but that this would not suggest that we read it "one hundred."

Draw still another tabulation form, this time writing hundreds, tens, ones.

hundreds	tens	ones

Request that the children count the ten tens with you again: "Ten, twenty, thirty, ... one hundred."

How many hundreds did we count just now? (One)

Enter a 1 in the tabulation chart.

If this set (point to the ten tens) were all we were counting, would we have any more tens to put in our chart? (No)

Would we have any ones in a remainder set? (No)

Enter zeros in the tens and ones columns. Ask the children if they can tell you how to write the numeral for one hundred. Write it to the right of the chart. (100)

Ask the children to help you count some more tens, by tens. Stop them at forty or fifty. Ask them how they would write the numeral for the number you have now counted, including the first ten tens on the chart. If anyone says 14 tens or 15 tens, enter it on the tens and ones chart and ask if there is another way to write it, so that you will know that you have already said "one hundred." (1 in hundreds column, 4 in tens column, 0 in ones column) Write the numeral to the right of the chart as before, and ask someone to read it. Point out that 14 tens and 140 name the same number. Go back, now, to the large set of tens, and have children help you count by tens. When you get to one hundred, hold up one finger, and go on, "one hundred ten, one hundred twenty, one hundred thirty, etc." to two hundred. Emphasize the two, and hold up another finger. "Two hundred ten, etc."

If children find it difficult to count by tens when they say "one hundred" or "Two hundred" first, go back and count again, this time just saying, when you reach the number, "one hundred, ten, twenty, ... two hundred, ten ... three hundred, ten," holding up another finger for each hundred as you reach it. When you arrive at the total number of objects counted, repeat the number clearly and ask the children how to enter it on the chart of hundreds, tens, and ones.

Point out that the first chart, showing 32 tens and 7 ones looks very much like the present one showing 3 hundreds, 2 tens, and 7 ones, and means exactly the same number of objects. Ask a child how to write the numeral as we would read it. (327)

Write three-place numerals on the chalkboard, have children tell what each part of the numeral means, and read the numeral. Be sure to include numbers like 401, 210, etc.; with zero, ones or tens.

Pupil Page 61: To show different ways of thinking about a number have children write the numerals, first showing hundreds, tens, and ones; second, in the usual way; third, as tens and ones.

Pupil Page 62(Optional): Top--have children draw a ring around the numeral for the greater number in each set. Bottom--Have children draw a ring around the numeral for the greatest number in each section.

Further Activities:

1. Children should have opportunities to count large collections of things and tell you how to write the number. This can be done in small groups or individually, and the length of time needed will depend on the sophistication (among other things) of your class. Theater tickets are easy to count into strips of ten, with the strips bundled to make hundreds. Hundreds-squared paper may be cut into strips of ten and counted the same way. In either case, however, you will need to point out that you are counting tickets or squares, not strips, to determine the number of objects, and that a strip containing ten squares is one ten, not just one thing.

2. Pass out cards on which 3-digit numerals are written. Ask each child to read his numeral, tell you how many hundreds, tens, and ones it means. Write it on the board after his name. When five or six have answered, ask which numeral names the greater number, this one or that one; which one names the lesser number, this one or that one; which one on the board names the greatest number. Enter greatest and least in a special part of the chalkboard, perhaps with the names of the children who read them. Repeat with other groups of children. When all have had a chance to report, and the numerals for the greatest and least numbers for each group have been recorded, determine which was the greatest number of objects counted and which the least.

3. When children are sure of the concepts desired here, let those who wish make booklets, using hundreds-squared paper, and write the numerals from 1 to 1000. Page 1 would be 1 to 100, page 2 from 101 to 200, etc. While not stressing the point, you may tell the children who finish the project (and many won't!) that the number after 999 is 1000, which means, as they can see from the 10 pages they have done, ten hundreds, or one thousand, zero hundreds, zero tens, and zero ones.

4. Use "SHOW-ME" cards, giving numbers orally, as "three hundred twenty-one", as "three hundreds, two tens, and one one," and as "thirty-two tens and one one."

Hundreds	Tens	Ones		Tens	Ones
3	3	2	332	33	2
4	6	8	468	46	8
2	0	0	200	20	0
1	0	5	105	10	5
6	9	5	695	69	5
5	7	0	570	57	0
9	1	6	916	91	6
1	4	8	148	14	8
	9	2	92	9	2
6	0	3	603	60	3
1	7	9	179	17	9
4	9	9	499	49	9

Draw a ring around the numeral for the greater of the two numbers in each set

154 [✓ 232]
[✓ 468] 399
[✓ 205] 186
[✓ 790] 437
617 [✓ 708]

219 [✓ 221]
[✓ 923] 687
[✓ 571] 479
813 [✓ 900]
455 [✓ 465]

Draw a ring around the numeral for the greatest of the three numbers in each set

[✓ 426] 321 108
647 784 [✓ 874]
260 [✓ 459] 381
98 107 [✓ 210]
186 445 [✓ 512]

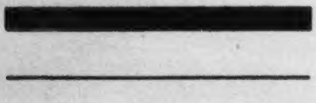
Chapter IX

INTRODUCING LINEAR MEASUREMENT

Background and Teaching Notes

This chapter will be concerned with line segments, with comparing and measuring line segments, and with the ruler. Our objective is to lead the child to understand what it actually means to measure the length of a line segment. This involves several distinct notions. First, we must understand what a line segment is. Second, we must have a means of comparing line segments to decide which is longer. (This is rather like the corresponding situation with numbers, where comparing sets is the notion that underlies the concept of number and of counting.) Third, we arrive at the notion of describing the length of a segment by using the number of copies of the unit that are needed to fill up the segment. Fourth, we finally arrive at the ruler as a convenient tool for finding this number.

Line Segments. The teacher may want to review the background for Chapter II, where the abstract nature of fundamental concepts such as point, line, and line segment is discussed. Of course the child's understanding of an abstraction must grow out of concrete experience, and at this stage we want to treat the ideas of line segment and point quite intuitively. On the other hand, we do not wish to teach ideas which must be later untaught. Thus a line segment is described as the path which is suggested to us by many of the objects of our physical space such as a tightly stretched string, the edge of a desk, a crack in the floor, a pencil, a straight mark on a paper, etc. We do not say that these things are line segments but that they make us think of line segments. They are pictures or representations or models of line segments. We do not try to emphasize directly that a line segment cannot be seen, but we hint at this by speaking of one thing being a better model of a segment than another. Thus a tightly stretched thread is a better model of a line segment than a tree, the edge of a desk is a better model than a piece of chalk, and the second of the pictures below is a better picture of a segment than the first.



A line segment has two ends and we refer to these as *points*. They are the *endpoints* of the line segment. Anything which reminds us of an endpoint of a segment, such as a corner of a floor tile, a dot on the chalkboard, etc. may be called a picture or model of a point. The smaller it is, the better picture it is.

Having introduced the term line segment, we might notice that some of the figures discussed before are formed of line segments. Thus a triangle is a closed figure formed from three line segments having common endpoints, and a rectangle consists of four line segments. They are what we earlier called the sides of these figures.

Comparison of Line Segments. One of the essential requirements for measurement of line segments is that two segments can be compared. That is, for any two line segments, we must have either the first longer than the second, or the same length as the second, or shorter than the second. This does not imply that we have yet learned to measure. In fact it is just the other way around, for without the idea of comparing there would be no way of discussing measurement. A similar situation occurs in consideration of weight where it is necessary to be able to compare two weights before one can describe what it means to measure weight.

A start on comparison of line segments was made in Chapter VI where squares were identified, though the term line segment was not introduced there. Here we need not only the idea of comparative lengths of line segments, but we also need a means by which a decision is to be reached.

It is quite easy to introduce a situation involving comparative lengths in which the answer is in doubt. This should lead to a recognition of the need for some means of making a decision. In most such cases the first suggestion made will be to place the objects side by side. In doing this, it will be well to emphasize frequently that the objects being compared are models of line segments. For example, we do not compare this book with that one. We compare the edge of this book with the edge of that one. In other words we wish length to be a property of segments, and this calls for frequent reminders.

The transition to the case when juxtaposition of objects is not feasible will occur almost automatically. The question as to whether the edge of the windowsill is longer than the edge of the door is of this type. Or a still simpler example would be which of two edges of a table is longer. In this problem situation it should be possible to elicit from the class suggestions as to how the comparison can be carried out. For example, one pupil may try holding his hands at the ends of one segment and transferring them to the other. Others may suggest using a piece of string or a strip of paper or a long stick which can be held or marked to match one segment and then carried over to lay against the other. In any case avoid any suggestion of measuring with a ruler, since it is the understanding of what measure really is that we are seeking to develop.

Measurement of Line Segments. The transition from the stage of comparison using an intermediate model to that of actual measurement is a small one but involves some added insights. Consider the question of going to the lumber yard to get a strip of wood as long as the edge of the desk. Clearly one procedure would be to find (or make) a model of wood or string or paper of the edge of the desk and ask at the store for a strip of wood that just matches this. Suppose, however, that there is nothing available which is as long as this desk edge. What now? Of course one could always take whatever objects were available, say a piece of chalk, a pencil, a soda straw, a short piece of string, etc. and put them end to end until he had something which just matched the edge of the desk. One could then put these objects in a bag, take them to the store, reassemble them end to end on the counter, and tell the clerk he wants a strip of wood that will just match this. Essentially one has still constructed a model of the edge of the desk, but it is one that may be more convenient since it can be taken apart and reassembled.

After the experience above, it might well occur to us that instead of hunting for just any miscellaneous collection of objects to form our model, it might be well to be systematic. Let us, for example, take a box of toothpicks and lay enough of these end to end to match the edge of the desk. Then we could pick up this set of toothpicks and, as before, take them to the store and reassemble them on the counter to indicate the desired length. Finally, it might occur to us to notice that the toothpicks are all the same lengths. It seems pointless to carry around a whole handful of identical toothpicks. Why not count the number of toothpicks and simply take along one as a sample? Then we can go to the clerk, show him the toothpick and tell him we want a strip of wood just as long as 14 of these laid end to end.

This last process is precisely the process of measuring a line segment. That is, to measure a line segment (here the edge of the desk) the first step is to select some convenient segment to serve as a unit. In this case the unit is the toothpick, or really a segment having the length of a toothpick. The second step is to place enough models or copies of this unit segment end to end to cover the segment being measured. The third step is to count the number of copies of the unit needed. The length is then described by giving the number and indicating the unit. Notice that measurement of a segment means giving a set of directions by which a segment can be constructed which just matches the given one.

The discussion of measurement should bring out three geometric insights.

a. Recognition that the segment selected as a unit is arbitrary; i.e. any segment may be used.

b. Realization that the number of unit segments necessary is different for different choices of the unit. Hence to properly describe the length of a segment one must not only give the number of units but indicate what the unit is.

c. Realization that most of the time measurement does not "come out even." That is, on occasion it will appear to take exactly 5 or exactly 7 copies of the unit segment to cover the segment being measured and in this case we say the length is 5 units or 7 units. Most of the time, however, we will find that, say, 4 copies of the unit segment are too few while 5 copies are too many. In this case all that can be said is that the length is between 4 and 5 units.

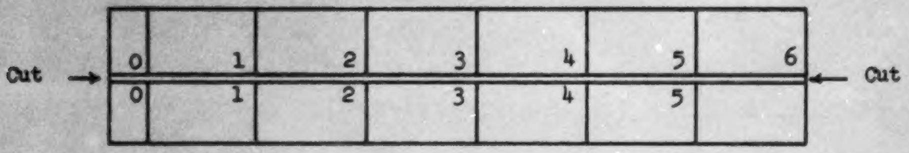
The Ruler. In the work above the pupil will have learned the meaning of linear measurement. That is, he will have measured by laying a model of the unit segment repeatedly along the segment to be measured. It is here proposed that he use this process of laying off a unit segment along a straight-edge to construct a measuring scale, that is, a ruler.

At this point we deliberately avoid introduction of standard units (such as inches) in order not to divert attention from the measurement process. Hence, the pupils construct their own rulers with their own units. The hope is that, if the pupil constructs his own ruler, he will better understand what it is. Also, in comparison with an ordinary commercial ruler, he will not have to ignore a lot of markings since the only marks on the ruler are the ones he placed there. Although care in working is to be encouraged, great technical skill in laying off the unit segment is not the point here. It is the idea of the construction of the ruler which is important, and as long as the ruler is sufficiently good to convey the idea, its technical lack of precision can be ignored.

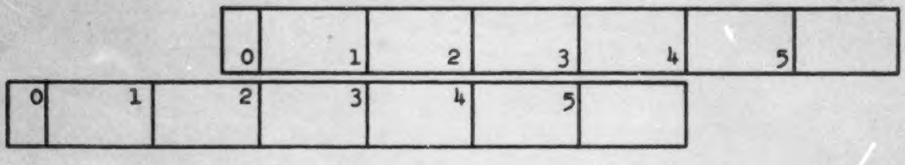
It is to be observed that the ruler is merely a model of a part of a number line, an idea that will have been introduced to the pupils informally earlier. This fact should be pointed out to the pupils.

The teacher may, of course, find it desirable eventually to discuss the inch as a standard unit which is commonly used by the children, but this should not be allowed to obscure the understanding of the essential arbitrariness of units of measure.

An application of the idea of the ruler to arithmetic which might be interesting to the more able children would be the construction of an addition slide rule. In essence it is merely a ruler made of, say, a light cardboard and then sheared down the middle as shown in the picture.


Cut
Cut
0
1
2
3
4
5
6
0
1
2
3
4
5

Numerals should appear on both the strips as shown. To add 2 and 3 using this device, slide the 0 of the upper scale until it is opposite 2 on the lower scale. The sum of 2 and 3 (i.e., 5) will then be found on the lower scale opposite the 3 of the upper scale as shown.


0
1
2
3
4
5
0
1
2
3
4
5

IX-1. Line segment-straightedge

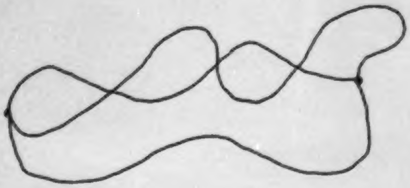
Objective: To introduce the concept of line segment and the use of the straightedge

Vocabulary: Straightedge, line segment

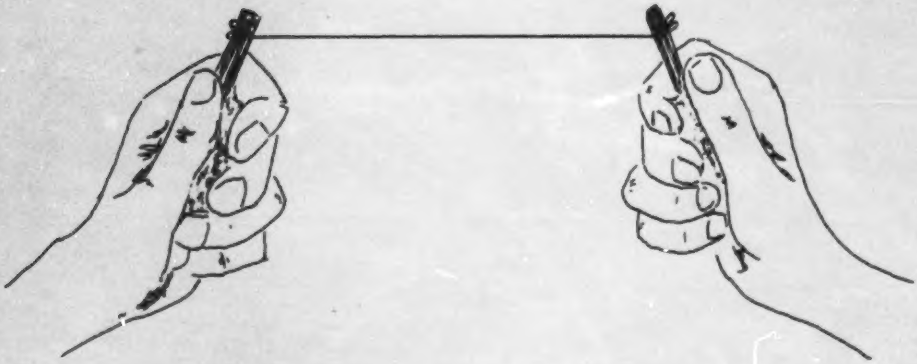
Materials: Jump rope, yarn, string, thread; various models of line segments; unmarked strips of cardboard (at least 10 inches in length)

Suggested Procedure:

Illustrate the idea of a path or route from one place to another by showing various positions of a string between two tacks, or the many different ways of drawing a path between two dots on paper or the board.



Discuss the path that would be taken if one were in a hurry. Pull a string tightly to suggest this idea visually.



Indicate that the path of the stretched string is called a line segment. Have children visualize the two ends called endpoints.

Ask the children to identify some objects that suggest to them line segments: the edge of a desk, pencil, piece of chalk, edge of a door, pointer, flagpole, telephone pole, edge of a bookshelf, edge of a sheet of paper, edge of a block, edge of a yardstick, straight mark on paper, etc.

Indicate that objects that suggest line segments are called models (or pictures) of line segments.

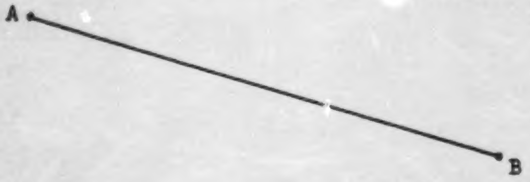
Emphasize that a segment has ends. Call attention to the physical things that suggest points. For example, if the edge of a block is mentioned as a model of a line segment, then the corners of the block represent the endpoints.

Discuss which of several models is the best model of a line segment. Use a jump rope, yarn, string, and thread for comparison. Help children begin to achieve the mental image that the thinner the model, the better the model makes us think of a line segment.

On the chalkboard show two points. Explain that these are models of points just as the objects were models of line segments.

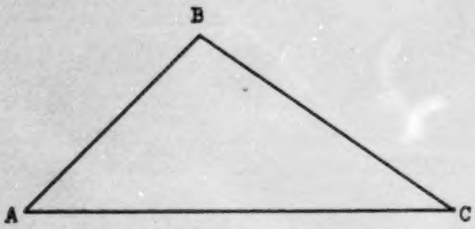
Draw a model of a line segment between the points. Use an unmarked cardboard straightedge, since one will be used later by the children.

Explain that it is often helpful to give names to the points. Label the points A and B as shown.



Explain further that the names of the endpoints may be used to name the line segment as either line segment AB or line segment BA. Illustrate and name several other line segments.

Uncover on the chalkboard a model of a triangle. Ask the children if there is a way in which they can use line segments and letters to describe the triangle. Then label the triangle.



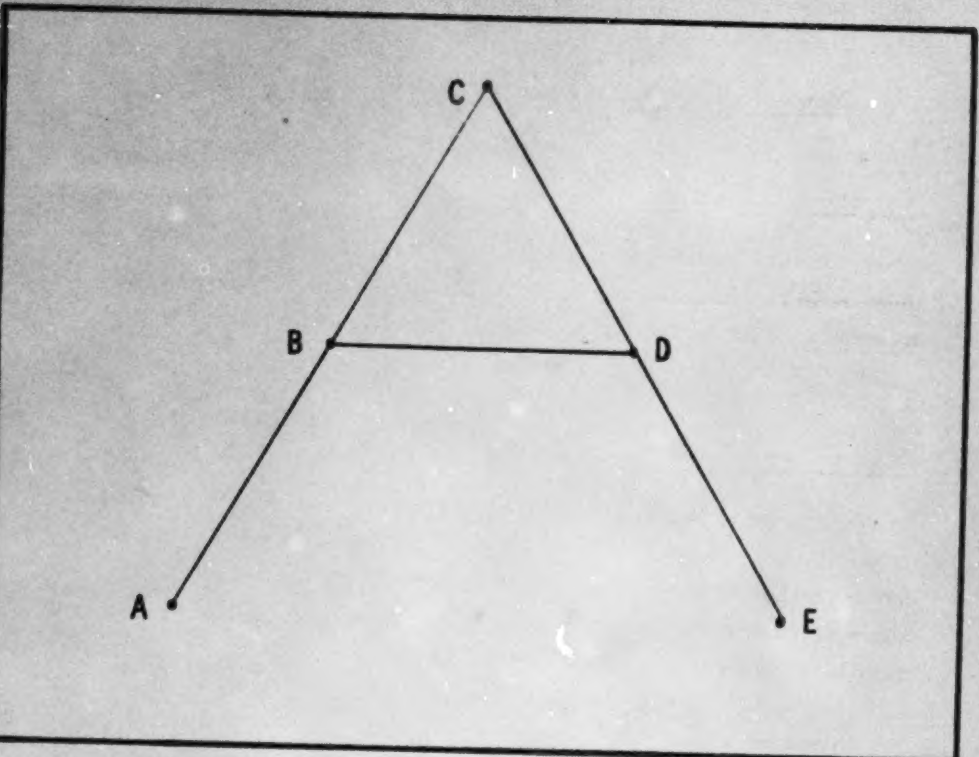
Help the children to visualize that this triangle can be described as being made up of line segment AB, line segment BC, and line segment AC.

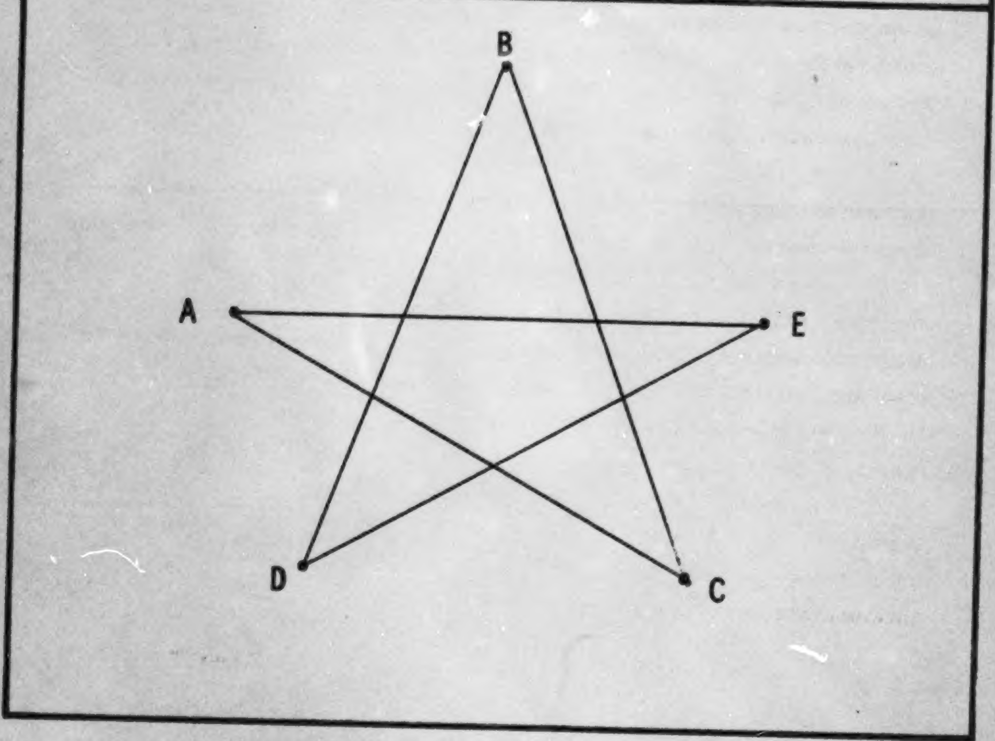
Ask several children to come to the board for a demonstration of the need to hold a straightedge firmly. Ask what will happen if fingers overlap the edge on which a segment is to be drawn. Have the children practice without necessarily using the chalk.

Distribute a cardboard strip to each child. Again give guidance as to the proper holding of the straightedge but do not let the instruction get in the way of the idea.

Pupil Page 63: Give oral directions to draw pictures of the segments AC, EC, and BD.

Encourage the children to guess what the lower figure will be. Draw in order segments AE, AC, BD, BC, and DE.


C
B
D
A
E


B
A
E
D
C

IX-2. Comparison of line segments

Objective: To compare line segments as a basis for the introduction of linear measure

Vocabulary: Review of length, line segment

Materials: String, straight stick (unmarked) longer than any of the segments to be compared, books, teacher chair, pupil chairs of slightly different height, flannel board

Suggested Procedure:

Review the idea of longer than, shorter than, and the same length as, in relation to the edge of a rectangle or square. Ask what the edges can now be called (line segments). In the review use illustrations where there is no obvious disagreement. The legs of the teacher's chair and a child's chair are typical models.

Now introduce a situation for comparison with children's chairs where there is no obvious agreement about height. A kindergarten chair or middle grade chair can be used if all chairs are the same height in your room. Tables of slightly varied height can also be used. Concentrate on the line segments rather than the objects as you proceed.

Ask how the lengths of the legs might be compared. Accept the suggestion that the chairs could be put back to back and any differences observed.

Use other illustrations within the classroom, such as books and pencils. Avoid children as objects for comparison since some may have been told they are "4 feet tall" or measured by the nurse, some may be embarrassed to be identified as shorter than another, and all make fairly "poor" models of line segments.

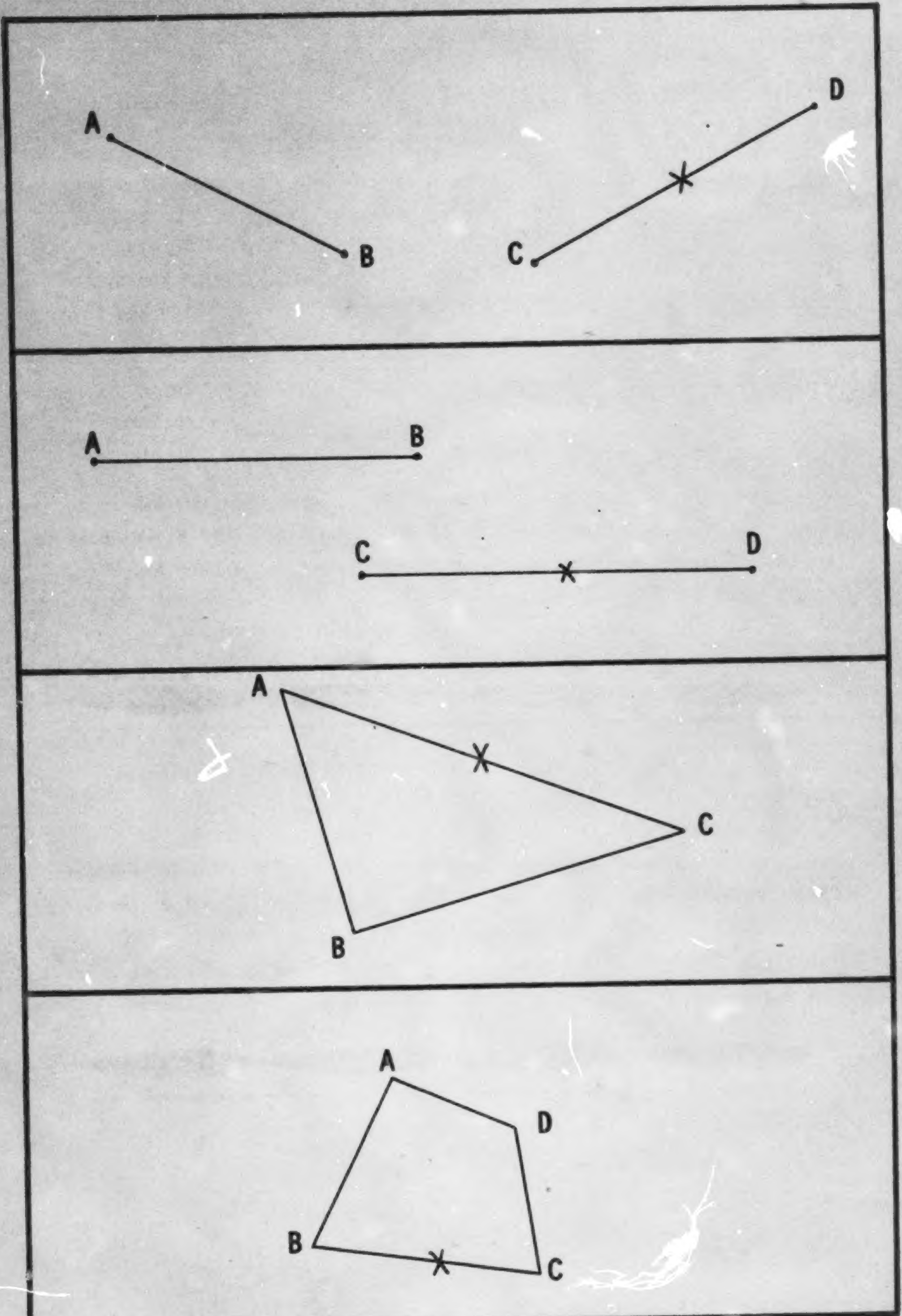
When there is understanding that some models of the line segments can be brought together for comparison, the transition to the need for an intermediate model can be made. Call attention to the two edges of the flannel board (one edge should be shorter). Ask how these models of line segments could be compared.

Accept any of the following ideas:

1. Holding the hands at the ends of one segment and using this to transfer to the other segment. (The end points are marked by the hands. Keep in mind that this method is quite imprecise and involves the notion of the line segment which is invisible.)
2. Marking the upper point of a segment on the body and using this to transfer to the other segment. (Remind the students that the line segment model runs from the floor up to a certain point of the body.)
3. Laying a piece of string beside one segment, and then grasping it carefully at the end points of the segment and carrying it over to the other segment. (Clarify that the string represents the segment, and the places where it is held the end points.)
4. Using a long unmarked stick by placing one end of the stick at one end of one of the segments, marking a point on the stick at the other end of the segment, and then comparing the marked stick with the other segment. (Indicate that the edge of the stick from one end to the mark represents the segment.)

Clarify that in each case above, in one way or another, a model has been made of one segment and this has been imposed on the other for comparison.

Pupil Page 64: Have children find the segment that is longer or longest and mark it with an X. Use a paper edge as a model of a segment.


A
B
C
D
A
B
C
D
A
C
B
A
D
B
C

IX-3. Measurement of line segments

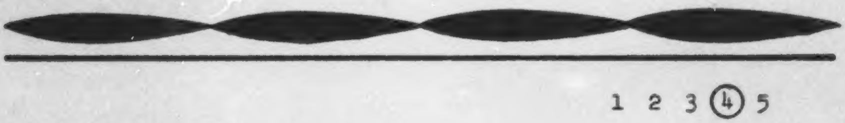
Objective: To introduce the idea of measurement of a segment as the number of unit segments necessary to cover it

Vocabulary: Unit segment

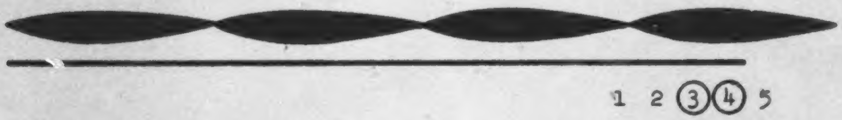
Materials: Toothpicks (for model units), pieces of drinking straws for same purpose

Suggested Procedure:

Provide each child with a number of models of a segment of some convenient length. (Toothpicks would be a reasonable possibility.) Make provision also, with drawings on paper, for a number of segments to be measured. The endpoints should be clearly indicated. The exercise is to see how many of these models (toothpicks) can be laid end to end along each segment.

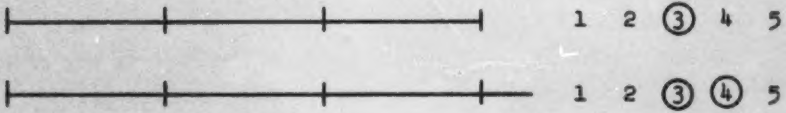


It is easy to arrange for answers to be indicated by drawing a ring as above. The length of the segment is then described as four of these units (here toothpicks). Either at this first experience or at least very early in his work the pupil should encounter problems like the following where the unit segment doesn't fit exactly.



Here three toothpicks are too few and four are too many so we describe the length as between 3 and 4 of these units. If this is the correct response, it can be indicated by putting a ring around both the 3 and 4 as shown. Of course any segment can be measured this way but the manipulative problems of laying the unit models end to end are such that it would be well to keep the mechanics as simple as possible to stress the ideas.

Now provide each pupil with only one model of the unit segment. Ask that it be placed repeatedly along the drawing and that a mark be made at the end each time so that the next measurement can be done carefully.



The problems themselves are like those in the preceding activity. One very common error in the first case above where the measurement came out "exactly" is in counting the number of marks the pupil has made instead of the number of times the unit segment was used. Problems could also be given in which the pupils use their unit to draw segments a certain number of units long.

Pupil Page 65: Lay models end to end along segments. Count units used. Draw a ring around one or two numerals to indicate the measure of the segment.

Pupil Page 66: Lay model repeatedly along segment. Make a mark at the end each time and count the marks. Draw a ring around one or two numerals to indicate the measure of the segment.

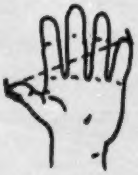
Further Activities:

Provide each child with several different units (say pieces of drinking straws) and have him measure the same segments with each unit. If straws are used there should be some designation attached to the different ones such as "long straw," "medium straw," and "short straw" so the pupil can describe his results as so many "short straws" etc. Another alternative would be to use different objects for models of unit segments, such as pencil, chalk, etc.

Have different pupils measure the same segment with different units. For example, have two children (with different sized feet) see how many of their foot lengths it takes to cover a crack in the schoolroom floor. (Possibly it would be even more impressive to have the teacher, who presumably has bigger feet, be one of the participants.)

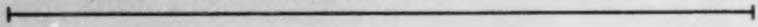
Have the pupils invent their own units and use them. For example, how many of some child's hand spans is it across the edge of the bookshelf?

In these exercises, try to make sure that the pupils keep clearly in mind that the unit is a segment. It is easy to have this obscured. For example, if a child is measuring using his hand span, it would be well for him to show on a sheet of paper a picture of his unit segment.

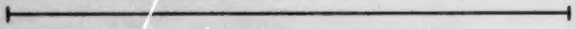


The exercises themselves will make clear the possible variety of units. Class discussion should crystallize the idea that for different units a measurement has different numbers so that to tell a length you need to tell not only the number of units but also to tell what unit was used. It should also be possible to develop the understanding that the smaller the unit the larger the number of them needed for any particular measurement.

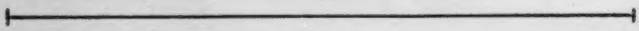
Answers will vary depending upon model selected for measuring. A uniform model for the class is useful.



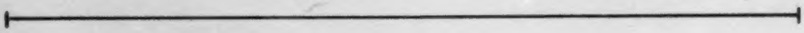
1 2 3 4 5



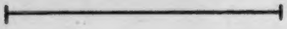
1 2 3 4 5



1 2 3 4 5

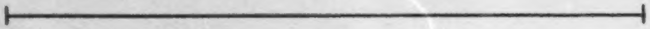


1 2 3 4 5

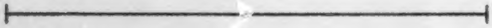


1 2 3 4 5

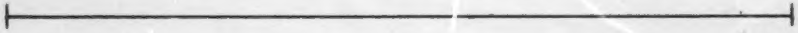
Answers will vary depending upon model selected for measuring. A uniform model for the class is useful.



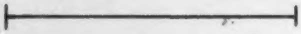
1 2 3 4 5



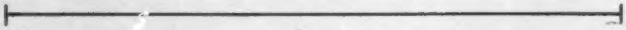
1 2 3 4 5



1 2 3 4 5



1 2 3 4 5



1 2 3 4 5

IX-4. Construction of a ruler

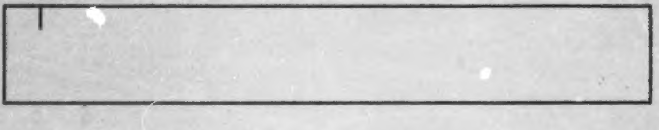
Objective: To introduce the idea of a scale as a measuring device

Vocabulary: None

Materials: Light cardboard straightedge (unmarked) perhaps a foot long, one for each pupil, models of some convenient unit segment (toothpicks or pieces of drinking straws) one for each pupil. To be convenient for handling, the units chosen should be around two inches or a little less.

Suggested Procedure:

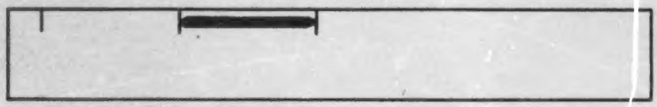
Distribute straightedges and ask each pupil to make a mark not far from the end. (This point is to be the zero point of the ruler. Note that the zero point is not at the end of the straightedge. In addition to being easier to identify, it avoids the problem that corners are always getting bent and dog-eared.)



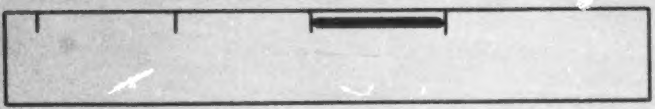
Now ask each child to put his model unit segment on the straightedge with one end on the initial mark and to mark the other end.

The piece of the straightedge now represents a segment one unit long.

Now the model unit segment can be laid down again,

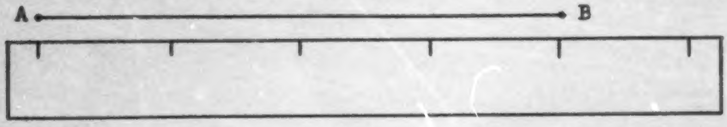


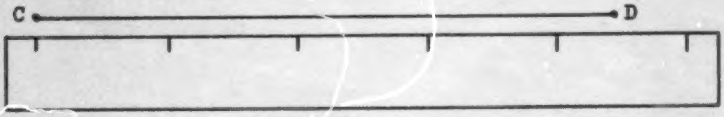
and again,



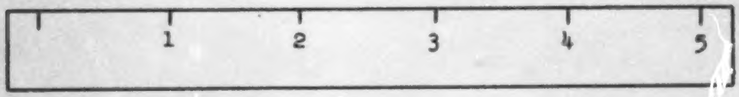
as often as the length of the straightedge allows. It is now easy to see that the marked straightedge shows models of segments 1 unit long, or 2 units long, etc.

The straightedge in its present form can now be used for measuring segments as shown below where it is seen that the length of line segment AB is 4 units and the length of line segment CD is between 4 and 5 units. These numbers are found by counting the number of unit segments. The placing of the straightedge will need to be emphasized, i.e., the placing of the original mark at one endpoint of the segment.

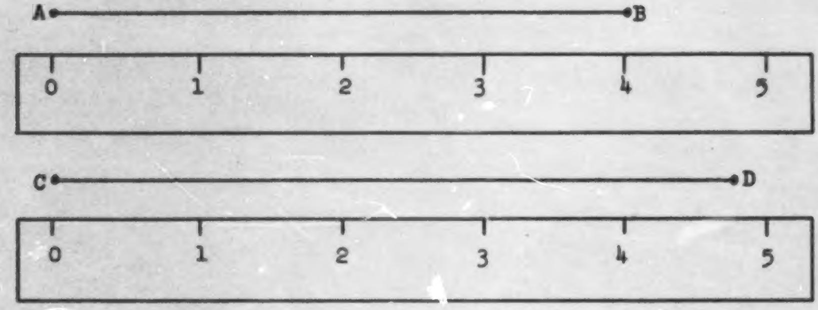




The next stage is to encourage the children to label the marks on the straightedge. The idea is to put a 1 beside the mark that was made the first time the model unit was used, a 2 beside the mark that was made the second time the model unit was used, and so on. The instrument then looks like this.



Discussion should produce the suggestion that the original mark be labeled 0. The instrument is now complete and may properly be called a ruler. Indicate that the ruler is just a model of a piece of a number line. In using it to measure line segments AB and CD as before, the numbering of the points produces a simplification.



The fact that in measuring line segment AB the point B is opposite the 4 mark shows that there were 4 copies of the unit segment between A and B. Thus instead of looking back and counting up the segments as was done before, the length of 4 units can be read directly from the ruler. Similarly the fact that for line segment CD, the point D is between the 4 and 5 mark shows the length of line segment CD is between 4 and 5 units.

Practice should be given in measuring with this device, but it need not be pushed too hard as the mechanics of using the ruler will be developed when standard units are introduced in a later grade.

Chapter X

EXTENDING ADDITION AND SUBTRACTION (6-10)

Background

The mathematical background for addition and subtraction has been discussed in some detail in Chapters IV and VII, and little more is needed for this chapter. We are principally concerned with extending the learning of addition and subtraction facts to include the addition facts through sums of 10 and the related subtraction facts range.

X-1. Visual perception

Objective: To develop visual perception of sets having six through ten members

Vocabulary: No new terms

Materials: Individual set material: beans, macaroni, jacks, cubes, etc.
Flannel board and flannel board materials
Cards with pictures of sets having 3 through 10 members

Suggested Procedure:

Children will have developed some facility for recognizing, without counting, sets with one through five members. Now they need to learn to use those sets in recognizing sets with more than five members. Suggestions are given in this lesson for developing visual perception of sets of 6 to 10 members.

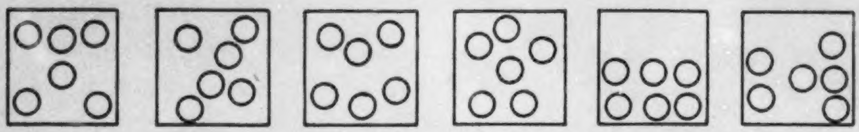
The first suggestions are given to help the teacher determine what sets the child already recognizes. Later lessons are presented to strengthen and extend the children's ability to use these sets.

Scatter 25 to 30 bottle caps on a table or on the floor. Direct the children to look for a set for which they can tell the number of members without counting. Ask a child to point to the set he sees. Make a ring around this set with chalk. Continue this activity until three or four sets have been indicated and marked. Do not continue until the sets are no longer easily identifiable sets.

Pupil Pages 67, and 68 : Direct the children to make a ring around any set for which they can recognize the number of members. Make the numeral for that set in the ring, too. (Do not expect each object to be in a ring.)

As a further activity, scatter a set of objects with 6, 7, 8, 9, or 10 members on a small rug or towel. Ask a child to pick up a subset which he can name immediately and begin his counting from that number. Each child should be encouraged to pick up the subset he identifies first rather than to look for a particular subset. Some children might pick up the first set and then recognize the remainder set without counting and then add the two numbers. Some children will also determine how many members in a set by recognizing three subsets and adding these three numbers.

The teacher can make cards that show sets of 6, 7, 8, 9, and 10 in a variety of arrangements. Each set should have a minimum of six arrangements so that children do not just learn a particular pattern. Sets should be constructed on the same color of card, preferably with the objects drawn alike to prevent association of a given set with a given object shape or card color.



Display the cards in the same way that the flannel board objects were shown in the earlier lesson. At times when this work is being done ask a child to "think out loud while you are finding out how many." Children may be encouraged to use these cards in an activity period on their own time and develop some of their own games that will help them learn to recognize these sets.

Give each child a set with 6, 7, 8, 9 or 10 members. Briefly display a card showing a set with 3 or 4 or 5 members. (The teacher should have several cards for each of these numbers.) Ask each child to look at his set and try to find a set with the same number of members. Display a different card and proceed as before. Then have each child find the number of members in his set by starting with a subset he can recognize without counting. After several cards have been used to look at the same set, the set each child is using can be changed by giving or removing one or more from his set.

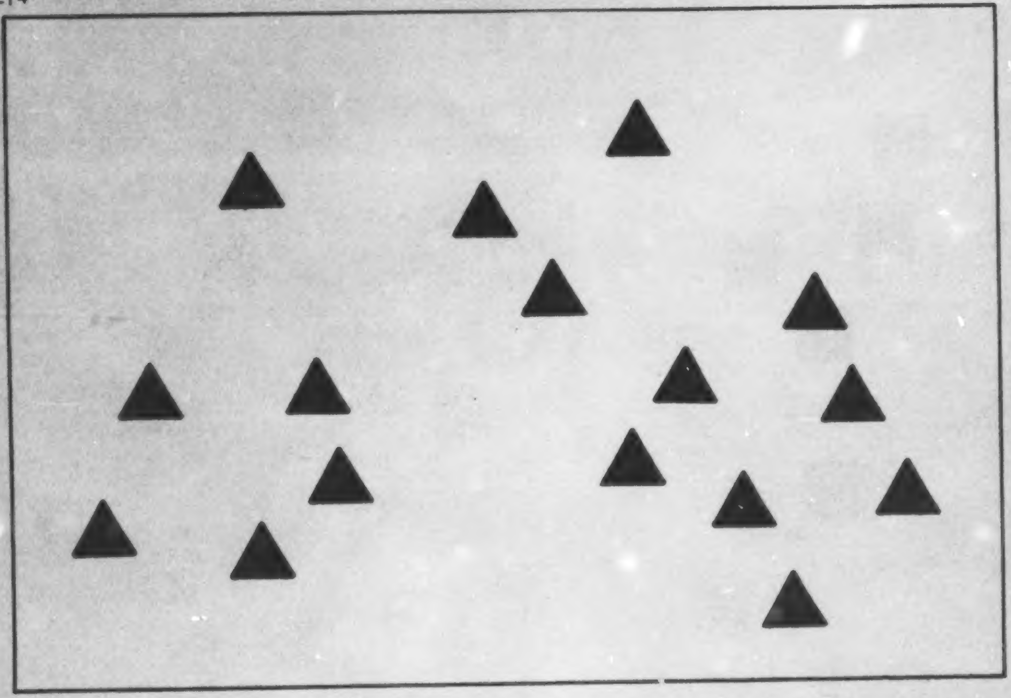
Pupil Pages 69, and 70: Direct children to look for one or more subsets, in the set in each box which are equivalent to the set at the left of the box.

Place a set of 8 objects on the flannel board. Ask a child how many are in the set. Ask the child how he knew there were eight in the set. You will probably receive one of several answers, such as, "I counted them." "There are 4 and 4 more." "I could see 5 and I counted the rest." Ask other children how they knew there were eight, until you have a variety of answers. Point out to children that even though we did it in different ways we all agree that there are 8 members in the set. Display a set with 3 members and ask a child to frame a set with just as many members within the set with 8 members. Ask the child to name that set "three" and begin his counting from there. Ask another child to find another set with 3 members within the 8 member set and count the whole set by continuing from the three. This will emphasize the point that which three were counted first does not make a difference in the result. Display a card with 4 members and repeat the experience. Display a card with 5 members and look for sets with this number of members in the set of 8. Help the children to see that the more members of the set we can see without counting, and yet know how many are there, the easier it is to find how many members are in the set. Help them to see that in different arrangements we will see different subsets. Continue this work with other sets. Use sets with 6 through 10 members at the outset so that children will not know immediately the number of members in the set.

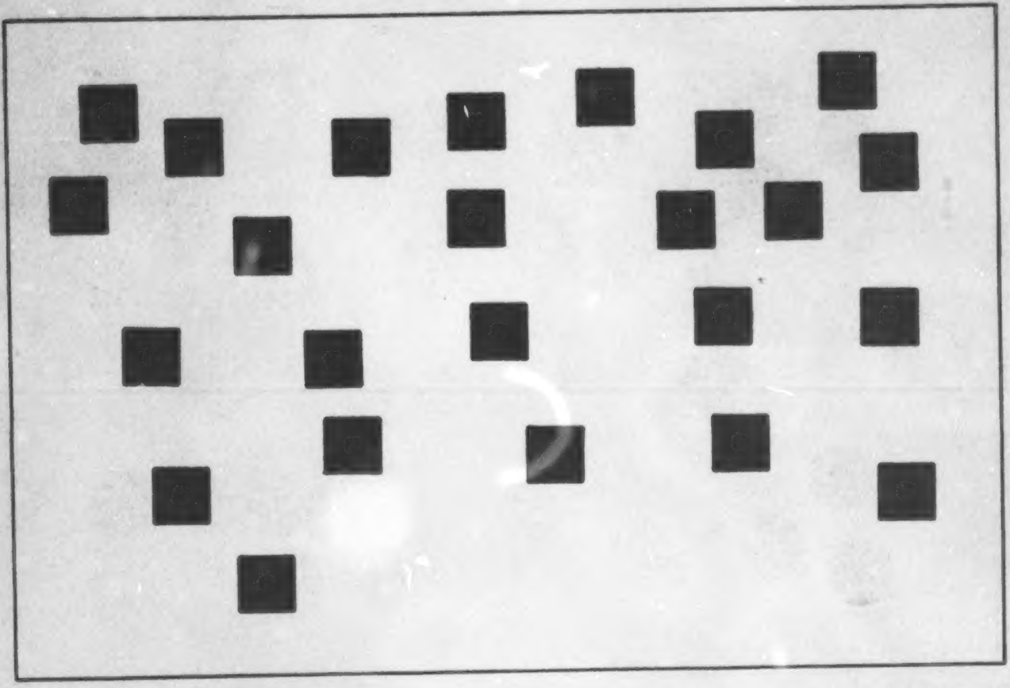
As a follow-up the teacher might use a small flannel board which can be concealed from the children while the set is being arranged and viewed only a short time before asking for a response. The danger in using a flannel board which is constantly in view is that children will be inclined to count the set as it is placed on the board. The intention of this activity should be to help them first look for a set they recognize and then count the rest of the set.

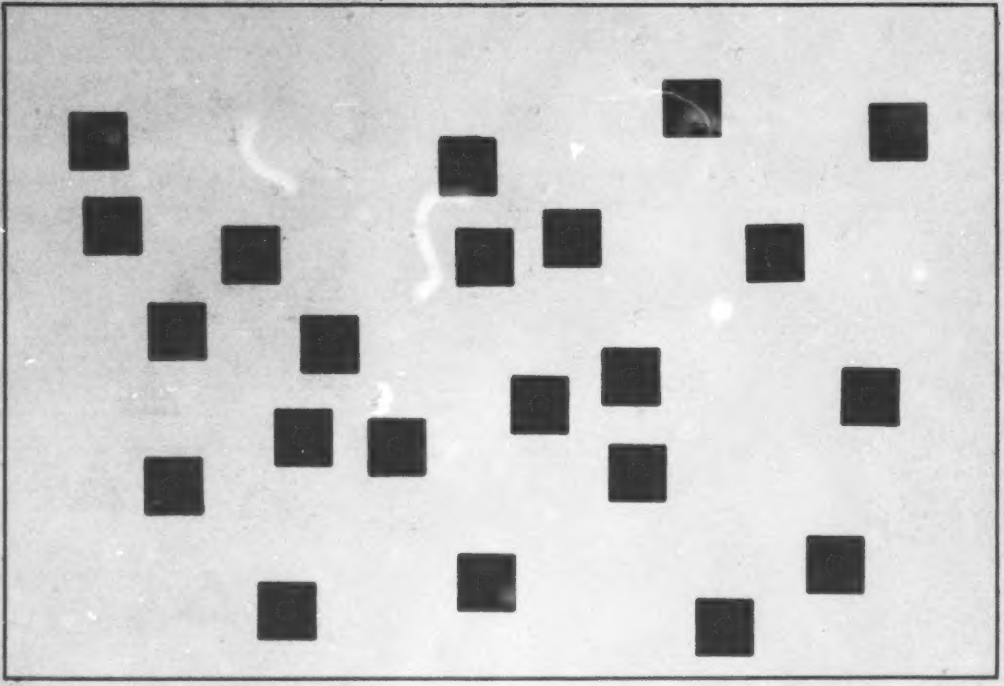
A higher level of perception is when children look for a subset they recognize and then recognize the rest of the set as another set for which they know the number of members. They then can add that number to the number of the first set rather than counting each member of the rest of the set. This is mentioned here only to make the teacher aware of this type of response, not to expect that all children will reach this level.

Pupil Pages 71, and 72: Direct pupils to ring a subset which they recognize and count the rest of the set. Write the total number of members in the set in the space provided.

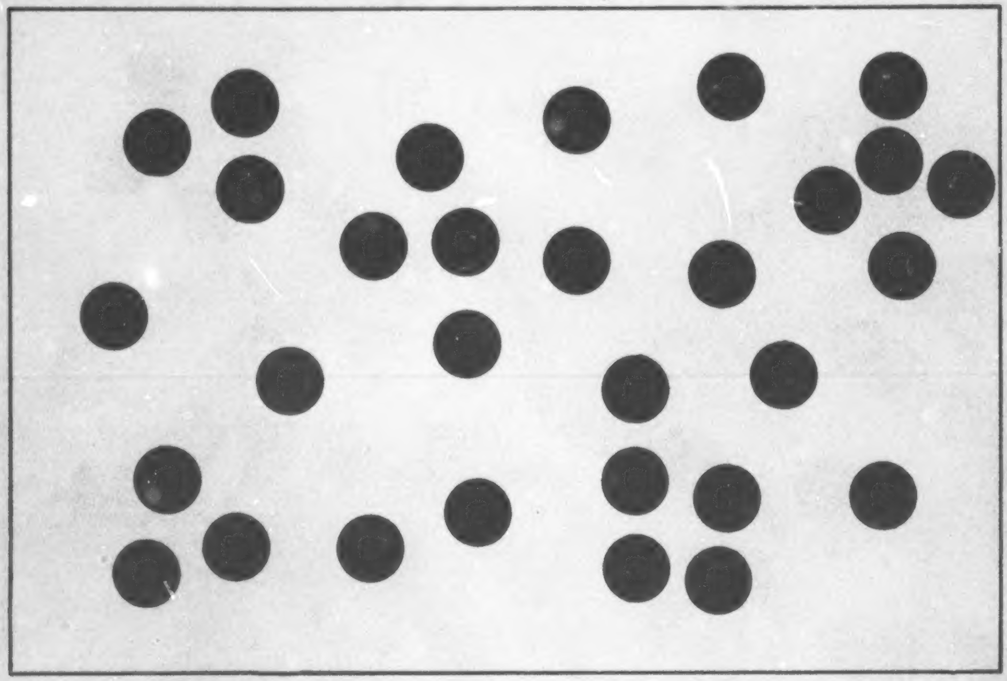


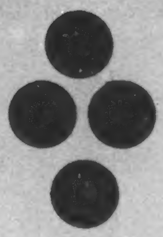
Answers will vary

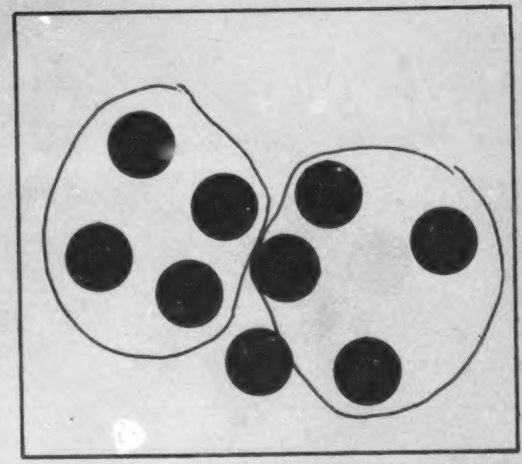


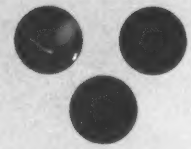


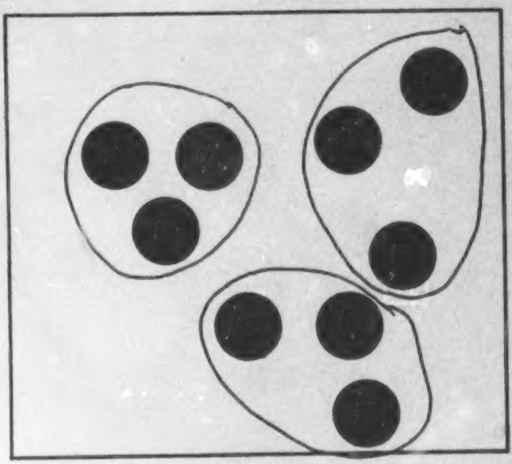
Answers will vary

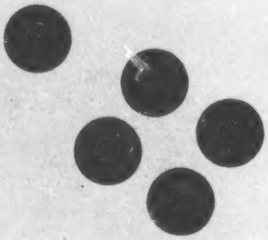


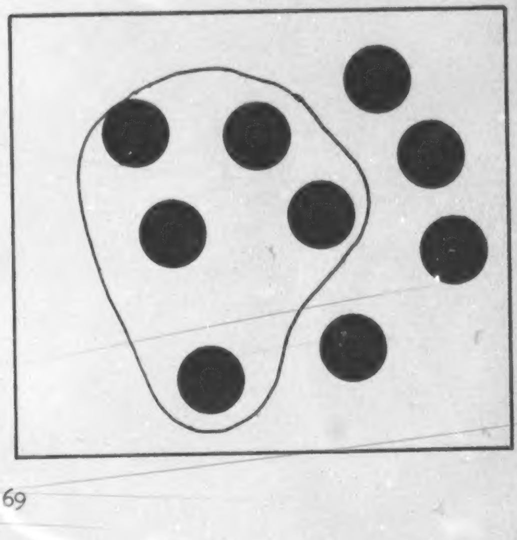


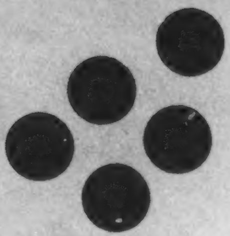


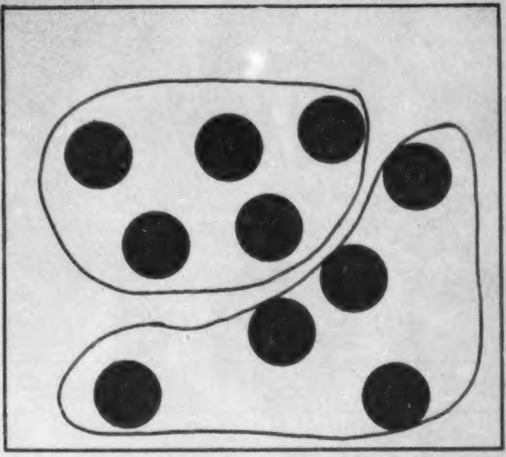


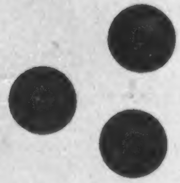


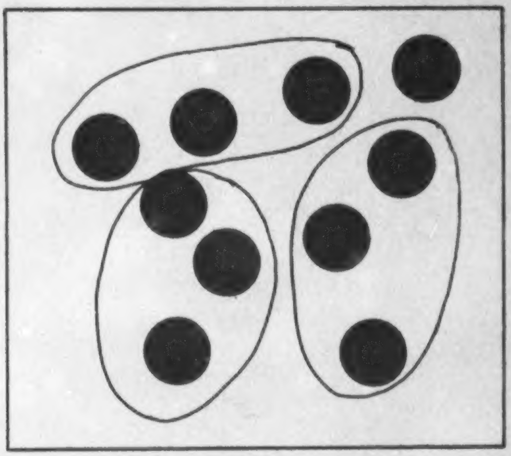


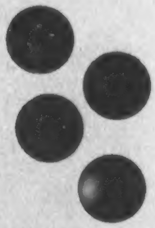


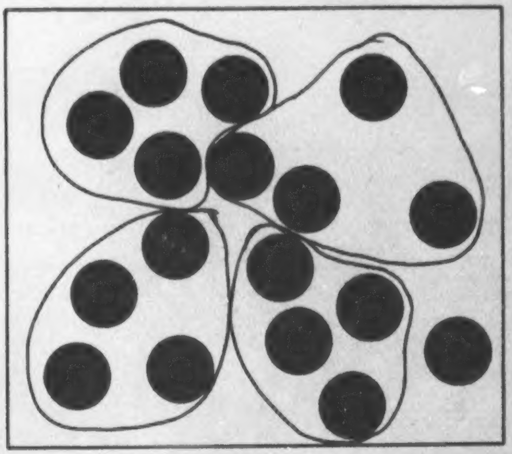


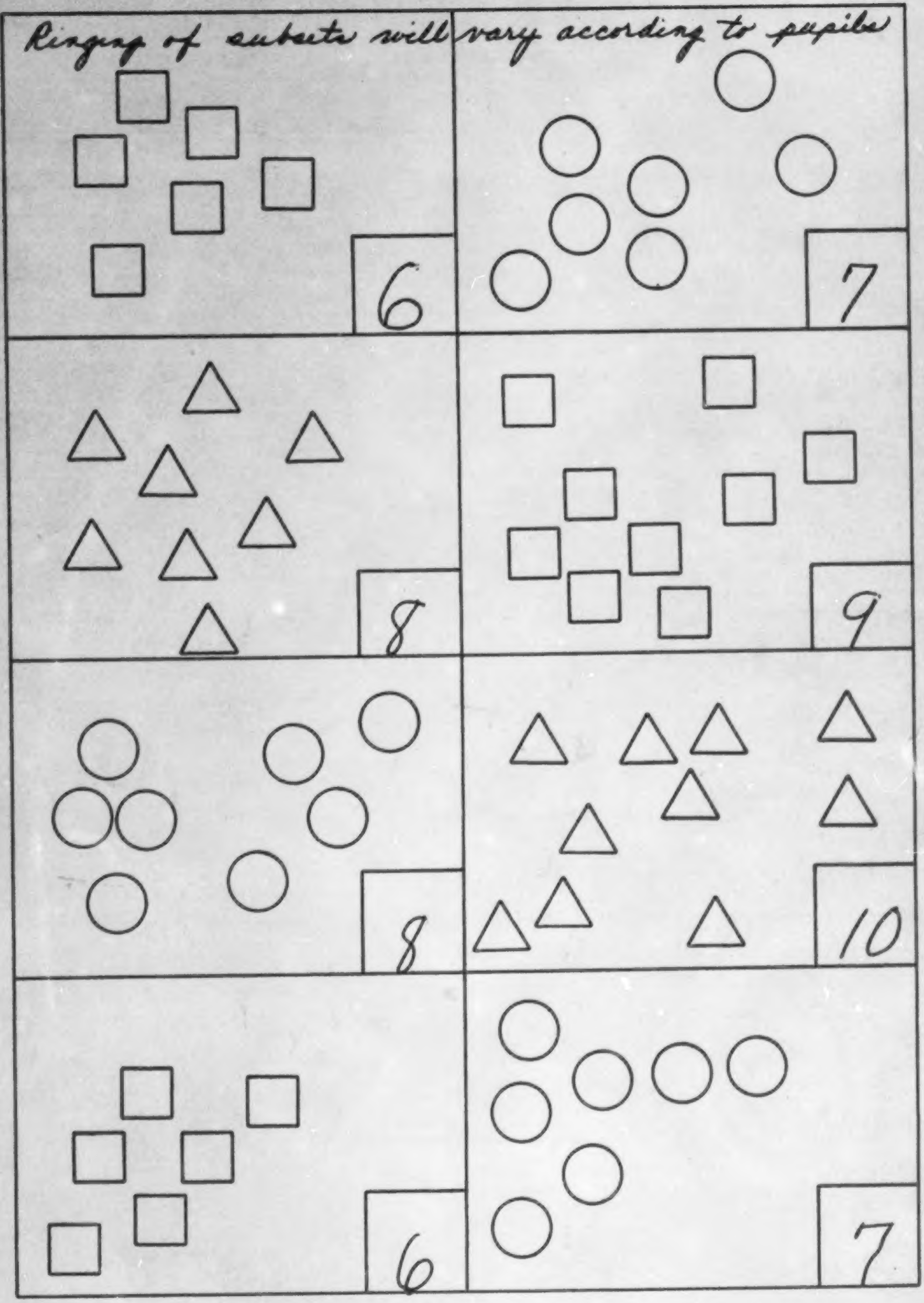


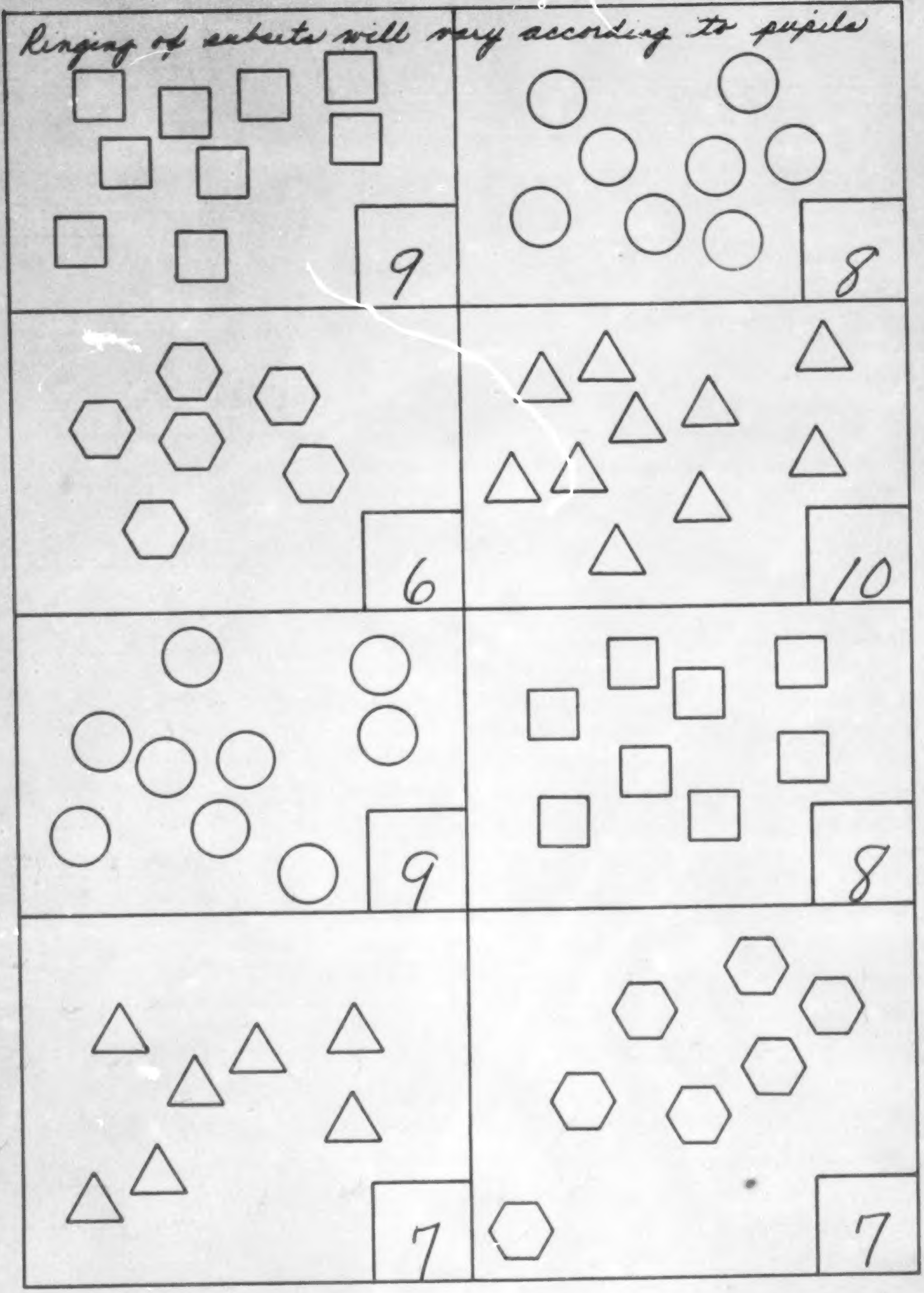









Ringing of subsets will vary according to pupils



Ringing of subsets will vary according to pupils

X-2. Partitions

Objective: To partition sets with 6 to 10 members
To relate addition and subtraction facts to this set operation

Vocabulary: No new terms

Materials: Individual set materials

Suggested Procedure:

Distribute to each child 6 to 10 objects. Ask the children to partition the set by putting the set of objects into any two subsets on their desks. (There will be many different partitions at this time.) Tell the children to pick up one of the subsets, thus removing it from the set.

> What set is left on the desk? (The other subset of the partition. A remainder set.)

Tell the children to join to the remainder set the subset they had removed.

> What set is on your desk? (The set we started with.)

Tell the children to make the same partition. This time tell them to remove the other subset.

> What set is on your desk? (A remainder set. The subset that we had removed last time.)

Tell the children to join the remainder set to the subset they had removed.

> Do we have the set we started with? (Yes)

Discuss with the children that removing either of the subsets of a partition leaves the other subset as the remainder set. Also note that joining the subset which had been removed to the remainder set results in the original set.

This game activity will give meaning to partitioning a set of objects.

Direct seven children to sit in a row at the front of the class. Tell these children to close their eyes. Then tap some of the children on the head. Then they are to open their eyes and those who were tapped should stand. Those standing and those still sitting will then be subsets of seven and the number of members in the subset can be counted.

Be certain to include instances in which all of the children will be tapped, or none of the children will be tapped in order to develop understanding of the partitioning which results in 7 and 0. Capable children may be able to realize very soon that they need to count only one of the sets because they will know the number of members in the other set.

Proceed to a more formal approach by giving each child a set with seven members.

Direct the children at this time to partition the set of seven so there are three members in one of the sets.

How many are in the other set? (4)

Tell the children that we can make a record of this partition by writing:

7 (can be partitioned) as 3 and 4.

Lead the children to make other partitions of 7 by telling them in each case how many members are to be in one of the subsets.

The completed chart would be:

7 as 3 and 4
7 as 2 and 5
7 as 0 and 7
7 as 1 and 6

Charts should be developed for sets with 6, 8, 9 and 10 members.

This time use the game activity, which was described earlier, to develop understanding of removing a subset of a partition. Tell the children who have been tapped to leave the set of seven children and form a subset. Thus the set of seven children will have had a subset removed. Those still sitting are the remainder set. This can be expressed with the equation $7 - 4 = 3$. Now ask the children to join the set of children who are the remainder set. This is the set we started with. This can be written as $3 + 4 = 7$.

Now ask the children who were not tapped to leave the group. The subset which is removed has 3 members. The equation is $7 - 3 = 4$. Ask the set of 3 children to join the set of children who are the remainder set. The equation related to this would be: $3 + 4 = 7$.

The equations for the partition of 7 as 3 and 4 are:

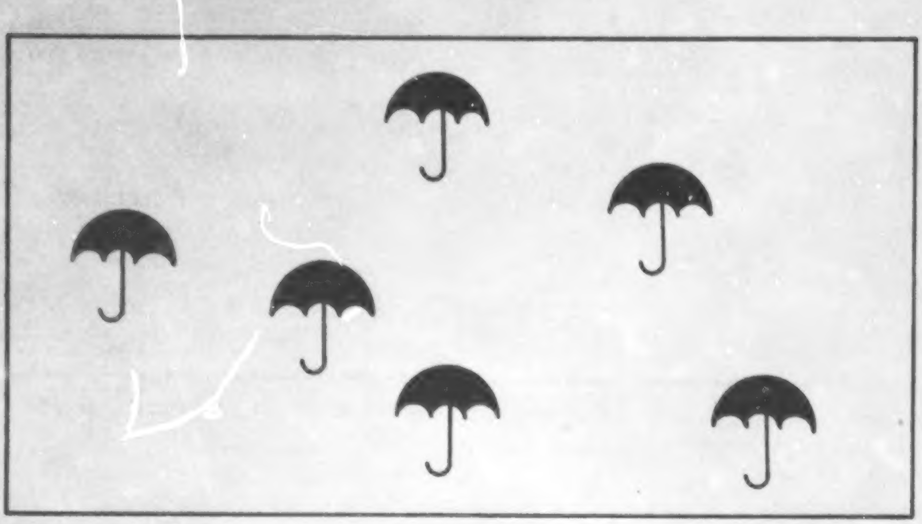
$7 - 3 = 4$	$4 + 3 = 7$
$7 - 4 = 3$	$3 + 4 = 7$

Continue by relating the addition and subtraction equations to all of the partitions which were developed in the earlier part of the lesson. So that the work is not a continual repetition of one activity, the teacher should vary the set activity used to develop the equations by including actual set experiences and imagined set experiences.

It is suggested that the partitioning of the sets into the various subsets will probably constitute one lesson for each set. The work involving the related subtraction and addition facts will probably take a lesson for each number. Work with the equations should not be started until the children are quite familiar with the partitions of each number.

Pupil Pages 73 through 77: Direct children to partition the set pictured at the top of each page by placing some object such as a strip of paper, a crayon, a piece of yarn, etc. across the box. Record the partition in the space provided at the bottom of the page. In some cases, not all lines will be filled.

Think of partitions of 6



6 as 1 and 5

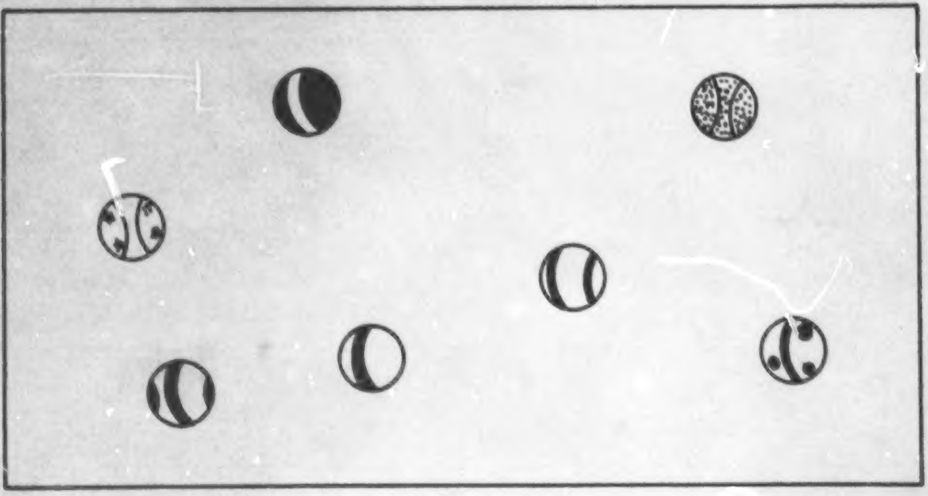
6 as 2 and 4

6 as 3 and 3

6 as 0 and 6

_____ as _____ and _____

Think of partitions of 7



7 as 1 and 6

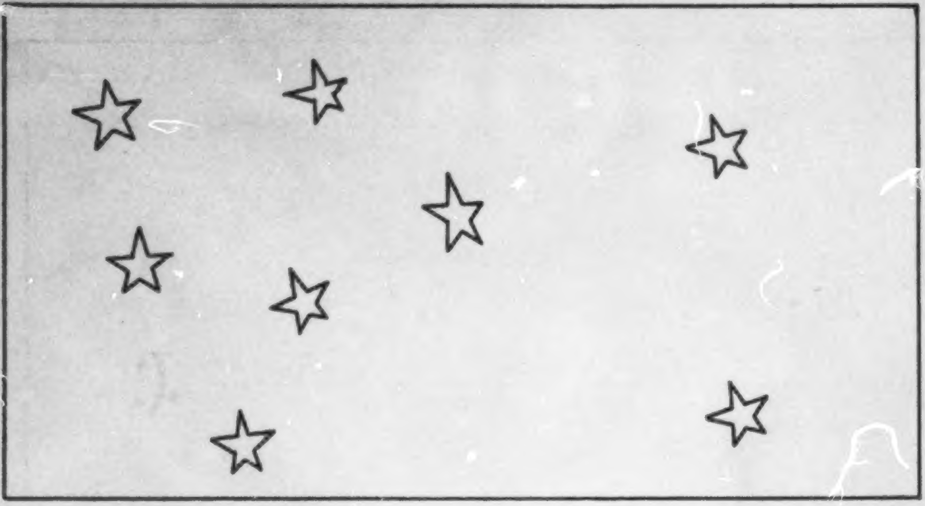
7 as 2 and 5

7 as 3 and 4

7 as 0 and 7

_____ as _____ and _____

Think of partitions of 8



8 as 0 and 8

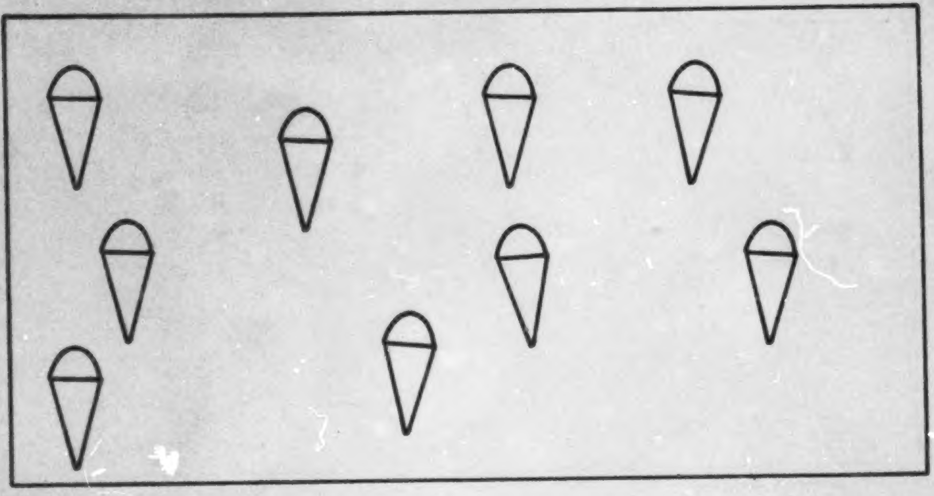
8 as 1 and 7

8 as 2 and 6

8 as 3 and 5

8 as 4 and 4

Think of partitions of 9



9 as 0 and 9

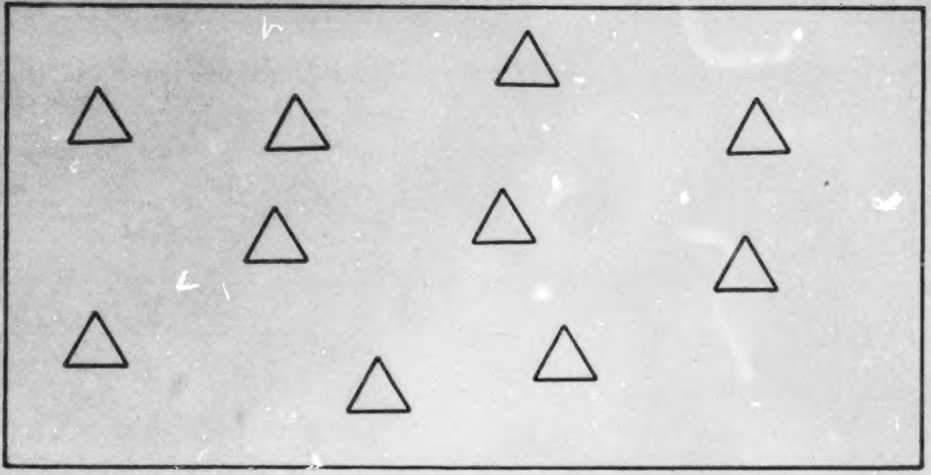
9 as 1 and 8

9 as 2 and 7

9 as 3 and 6

9 as 4 and 5

Think of partitions of 10



10 as 0 and 10

10 as 1 and 9

10 as 2 and 8

10 as 3 and 7

10 as 4 and 6

10 as 5 and 5

X-3. Joining and removing

Objective: To extend joining and removing sets to include 6 to 10 members
To deepen understanding of the number facts by developing them through imagined joining and removing of sets

Vocabulary: No new terms

Materials: Flannel board, flannel board objects

Suggested Procedure:

Place on the flannel board a set of numeral cards 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and the symbol cards - and =. (3" by 5" file cards are a good size.) Using another set of cards the teacher places an equation on the flannel board. The sum of the equation will be represented by a blank card.

$$\boxed{3}\ \boxed{+}\ \boxed{5}\ \boxed{=}\ \boxed{}$$

Ask a child to place on the flannel board sets about which the equation helps him think. Next join the set with 5 members to the set with 3 members. What is the number of members in the set? Replace the blank card with the numeral card which completes the equation.

Several of these experiences may then be followed by giving the children set materials and individual sets of numeral cards. The teacher places an equation with a missing sum on the flannel board. The children select the card which completes the equation from their own set and places it in a designated place on his desk, i.e., you may want them to place the card upside down at the upper right corner of their desk. The teacher can then check several of the cards and show the correct response or she may wish to have all children hold their cards up. If necessary, this should be followed by having a child show on the flannel board how he used set materials to solve the equation. Set materials should be on the desks of all children and those who need to use them should be encouraged to do so. It should not be necessary, however, for all children to use set materials.

Follow this activity by presenting imagined set experiences and placing the appropriate equation on the flannel board with the numeral and symbol cards. Some examples of imagined set experiences might be these. (Encourage children to give problems.)

Some statements should be made about which an equation cannot be written because of the lack of information or the information given is irrelevant to the problem. A few such statements are given here as illustrations.

Bill has 6 cookies.
Jim has 3 cookies.
How many do they have together? $(6 + 3 = \square)$

Jean has 8 kittens.
Don gave her 2 more.
How many kittens does she have? $(8 + 2 = \square)$

Jan has 2 red books in her desk.
She has some blue books in her desk, too.
How many books does Jan have in all? (No equation)

Jack has 7 marbles.
Mother gave him 3.
How many marbles does he have now? $(7 + 3 = \square)$

To implement work with removing a set have the children imagine a birthday cake with 6 to 10 candles. For illustrative purposes 8 will be used in the discussion here. Place numeral cards 0 through 8 in a box. Draw a card from the box and show it to the children. Tell them this is the number of candles which you were able to blow out with one try.

Discuss the number of candles which would still be burning. (If necessary, a row of candles, with detachable flames might be placed on the flannel board. When the number which indicates how many are blown out is shown, that number of flames could be removed.) Each experience should be followed by writing an equation.

A chart could be made about these experiences to show all the different equations which were formed. Then when a child draws a card which has been drawn earlier he could be asked to find the equation which represents this. Only the different equations would be added to the chart.

This should be followed by giving imagined set experiences about which the children will form equations. Use the flannel board numeral and symbol cards to do this. When the children can write no equation, discuss the reason.

Mother made 6 cup cakes.
Billy ate 2.
How many were left? (6 - 2 = 4)

Billy had 4 goldfish.
He gave some to Jamie.
How many does he have left? (No equation)

There were 9 books on the table.
Joe took 5 books to his desk.
How many books were left on the table? (9 - 5 = 4)

Carol had 7 pieces of candy.
She ate 2 of them.
How many pieces were left? (7 - 2 = 5)

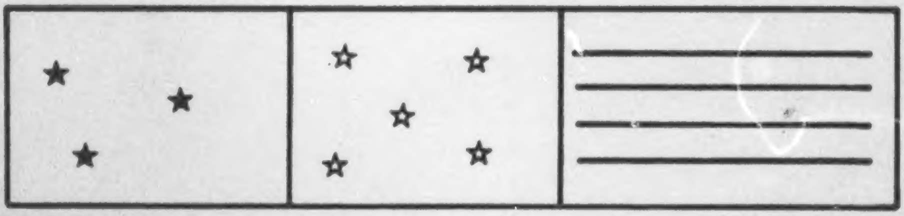
Mike had 6 cars.
He gave 3 to Julie.
How many boats does Mike have? (No equation)

To further understanding of the relationship between addition and subtraction, place a set of 2 objects on the flannel board, and then a set of 6 objects near it. Record on the chalkboard the number of members of each set, and the number of members of their union (but do not join the sets). Record this information in the form:

6	2	8

Ask the children for two equations for these two sets and their union. By asking pupils to visualize joining the set of 2 to the set of 6, and then asking them instead to think of joining the set of 6 to the set of 2, try to elicit the equations: $6 + 2 = 8$, $2 + 6 = 8$. Then ask them to think of the set of all eight objects on the flannel board, and think of removing one of the subsets into which it is separated. Try to elicit the equations: $8 - 2 = 6$, $8 - 6 = 2$. Point out that from the single arrangement on the flannel board we can see that all four of the following are true: $6 + 2 = 8$, $2 + 6 = 8$, $8 - 2 = 6$, and $8 - 6 = 2$.

Continue this sort of activity. Begin with two sets. Think of their union and the corresponding equations involving addition. Then think of removing one of the sets from the union to elicit the corresponding subtraction facts. This sort of activity is continued with the Pupil Page 78. The teacher should begin the work of the first sheet with a chalkboard display, perhaps as follows,



Pupil Page 78: Think about joining, and then about removing, and then write the equations that are suggested by these actions.

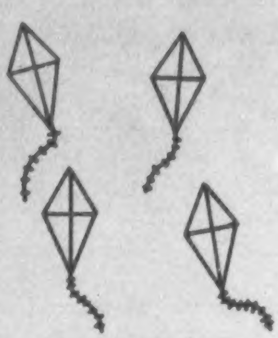
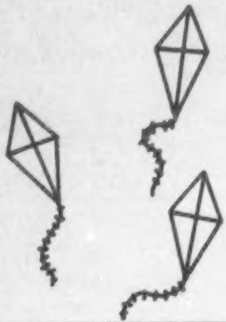
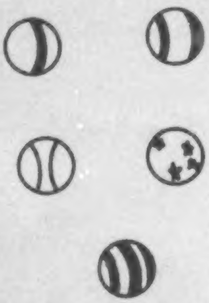
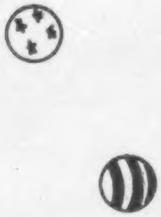
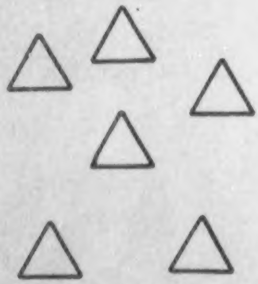
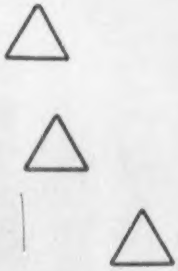
The next stage of the development should also begin with a flannel board demonstration. Begin with an equation, say $3 + 7 = 10$. Then ask a child to make on the flannel board a picture that this equation suggests to him. Either a set of 3 and a set of 7, or a set of 7 and a set of 3, forms a suitable picture for the next part of the development. Finally, ask the children what other equations this picture suggests to them. Elicit all four of the equations: $3 + 7 = 10$, $7 + 3 = 10$, $10 - 7 = 3$, and $10 - 3 = 7$.

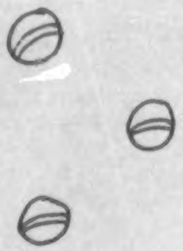
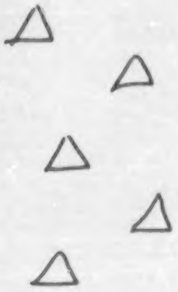
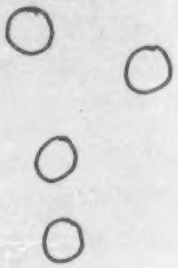
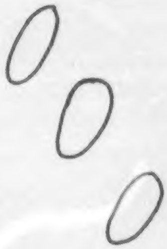
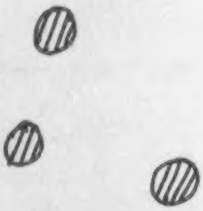
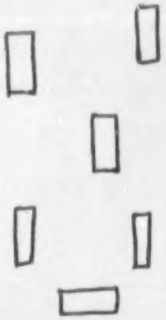
Pupil Page 79: Make a picture that each equation suggests. Then write other equations that the picture suggests.

The final stage of this development is reached when drawing the picture of the sets is replaced with imagining the sets. The following sort of introduction might be made:

> Let us think about this equation:
>
> $5 + 4 = 9$ (on the chalkboard)
>
> What sort of picture would go with this equation?
> (Elicit a description of sets, say a set of five balloons and a set of four balloons.)
>
> Very well. Let us imagine a set of five balloons here (pointing to a place on the chalkboard) and a set of four balloons here, so that there are nine balloons in all. Does this picture that we are imagining suggest any other equations? Remember, we have five balloons here, four balloons here, and nine balloons in all. What equation do you think of? (Try to elicit: $5 + 4 = 9$, $4 + 5 = 9$, $9 - 5 = 4$, $9 - 4 = 5$.)

Pupil Page 80: Imagine sets which illustrate the equation, and then record the other equations.

Sets		Equations
		4 + 3 = 7 3 + 4 = 7 7 - 4 = 3 7 - 3 = 4
		5 + 2 = 7 2 + 5 = 7 7 - 5 = 2 7 - 2 = 5
		6 + 3 = 9 3 + 6 = 9 9 - 6 = 3 9 - 3 = 6

Sets		Equations
		$3+5=8$ $5+3=8$ $8-5=3$ $8-3=5$
		$4+3=7$ $3+4=7$ $7-4=3$ $7-3=4$
		$3+6=9$ $6+3=9$ $9-6=3$ $9-3=6$

Equations

5 + 4 = 9 4 + 5 = 9 9 - 5 = 4 9 - 4 = 5	9 + 0 = 9 0 + 9 = 9 9 - 0 = 9 0 - 9 = 0
8 - 6 = 2 8 - 2 = 6 6 + 2 = 8 2 + 6 = 8	7 - 7 = 0 7 + 0 = 7 0 + 7 = 7
7 - 2 = 5 7 - 5 = 2 5 + 2 = 7 2 + 5 = 7	4 + 6 = 10 6 + 4 = 10 10 - 6 = 4 10 - 4 = 6

X-4. Missing numbers in equations

Objective: To solve equations in which the missing number is a sum, a difference, or one of the pair of numbers needed to determine the sum or difference

Vocabulary: No new terms

Materials: None

Suggested Procedure:

Review with the children Unit VII, Lesson 7, in which the missing numbers are in a variety of places in the equation. Ask the children what numbers we are trying to find in each of the following equations.

$6 - \underline{\ 1\ } = 5$ (What number subtracted from 6 equals 5?)

$5 + \underline{\ 4\ } = 9$ (What number added to 5 equals 9?)

$3 + 4 = \underline{\ 7\ }$ (4 added to 3 equals what number?)

$8 - 7 = \underline{\ 1\ }$ (7 subtracted from 8 equals what number?)

Present many equations with a missing number in one of these places and have the children use actual set materials to complete the equation. Present imagined experiences. Have children write the equation which would be written about the problem and then solve it.

Some statements should be presented for which equations can not be written. These are given to help children begin to realize when more or different information is needed to solve a problem.

Examples of imagined experiences are given here:

Mary had two balls
Mother gave her some.
Now she has seven.
How many did her mother give her? $(2 + \underline{\ 5\ } = 7)$

Tom had six boats.
He gave some to Bill.
Now Tom has five boats.
How many did he give to Bill? (6 - 1 = 5)

Gordy has 5 guns.
Eddie has 7 guns.
How many more guns does Gordy have? (No equation)

Ben has 8 marbles.
Three are red.
The rest are green. (8 = 3 + 5)
How many are green? (3 + 5 = 8)

Mark had three cookies.
He ate some on the way to school.
Now he has only one for lunch.
How many did he eat on the way to school? (3 - 2 = 1)

Karen had three caps.
Her friend gave her five caps.
How many caps does Karen have? (3 + 5 = 8)

Bryan bought some ice cream cones.
Four were vanilla and the rest were strawberry.
How many did he buy in all? (No equation)

Jack's father gave him three model rockets.
His grandfather gave him five.
Jack gave one of them to his friend Douglas.
How many rockets does Jack have? (3 + 5 - 1 = 7)

Pupil Pages 81-83: Direct children to fill in the missing numerals in these equations. Some children may need to use set materials.

$3 + \boxed{4} = 7$

$4 + \boxed{5} = 9$

$1 + \boxed{4} = 5$

$9 + \boxed{1} = 10$

$7 + \boxed{0} = 7$

$6 + \boxed{2} = 8$

$2 + \boxed{4} = 6$

$0 + \boxed{6} = 6$

$5 + \boxed{4} = 9$

$10 - \boxed{3} = 7$

$7 - \boxed{1} = 6$

$8 - \boxed{3} = 5$

$9 - \boxed{7} = 2$

$7 - \boxed{6} = 1$

$6 - \boxed{0} = 6$

$10 - \boxed{2} = 8$

$9 - \boxed{5} = 4$

$6 - \boxed{3} = 3$

$5 + \boxed{2} = 7$

$10 - 2 = \boxed{8}$

$7 - \boxed{7} = 0$

$1 + \boxed{9} = 10$

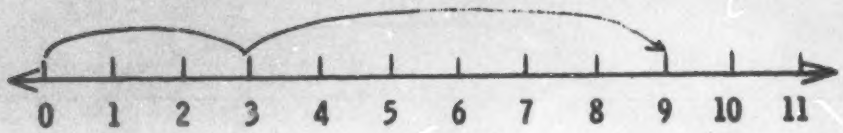
$0 + 8 = \boxed{8}$

$4 - 1 = \boxed{3}$

$9 - 3 = \boxed{6}$

$8 - \boxed{3} = 5$

$6 + \boxed{2} = 8$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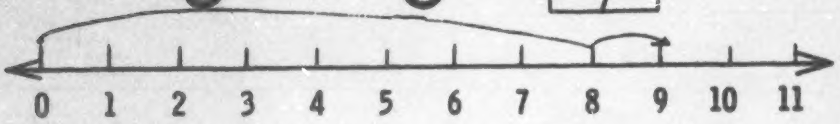
$3 + 6 = \boxed{9}$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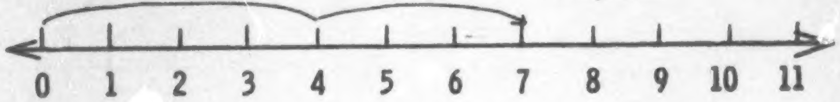
$8 + 1 = \boxed{9}$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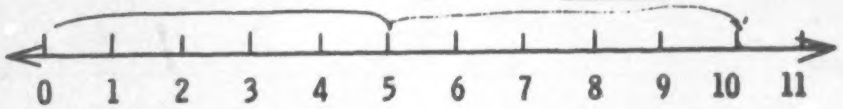
$4 + 3 = \boxed{7}$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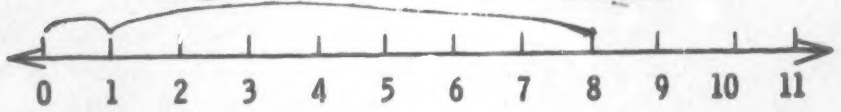
$5 + 5 = \boxed{10}$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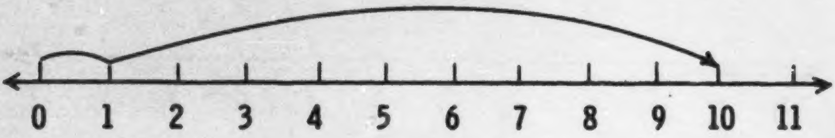
$1 + 7 = \boxed{8}$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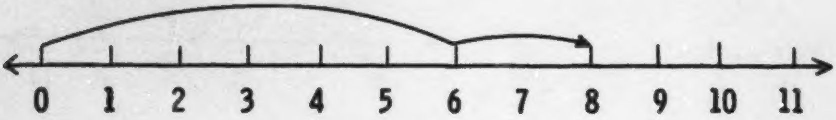
1 + 9 = 10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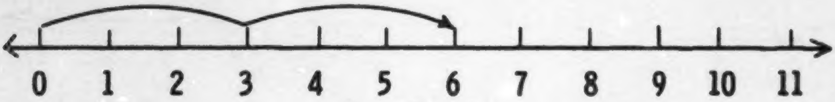
6 + 2 = 8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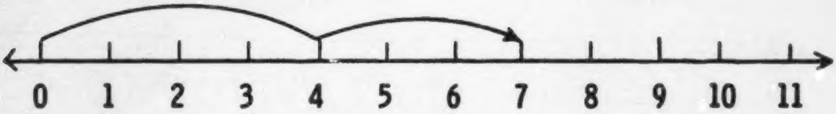
3 + 3 = 6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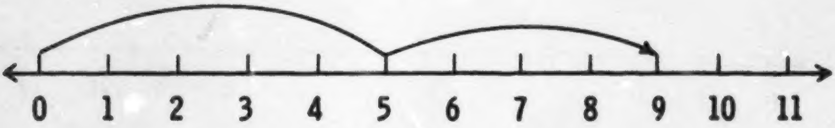
4 + 3 = 7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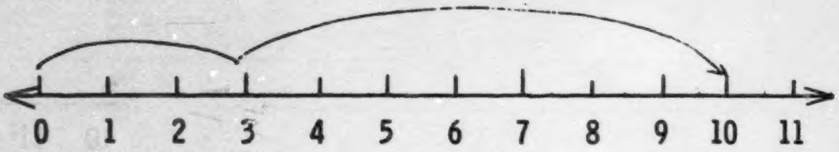
5 + 4 = 9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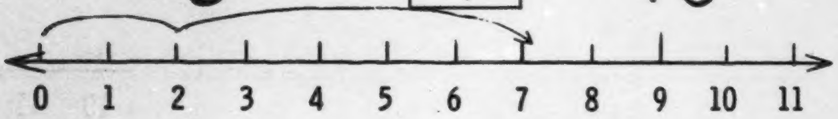
$3 + \boxed{7} = 10$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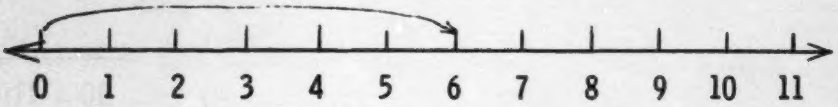
$2 + \boxed{5} = 7$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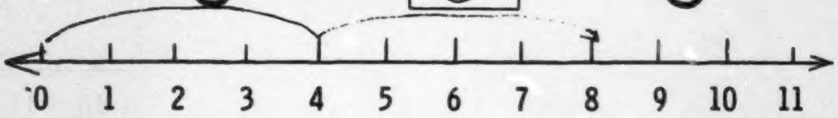
$6 + \boxed{0} = 6$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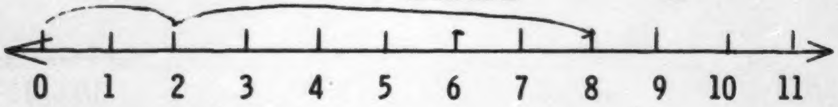
$4 + \boxed{4} = 8$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$2 + \boxed{6} = 8$

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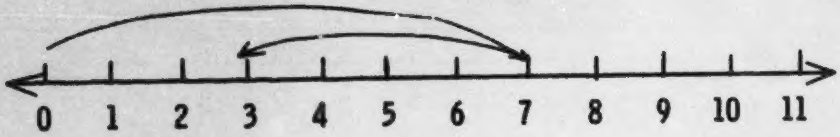
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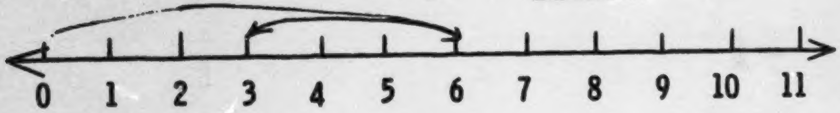
$$7 - 4 = \boxed{3}$$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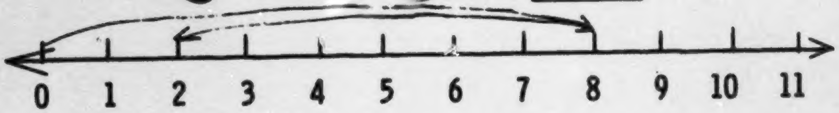
$$6 - 3 = \boxed{3}$$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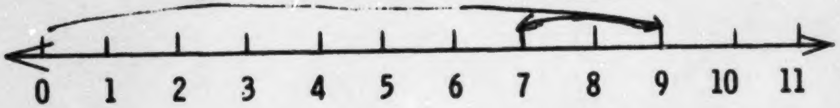
$$8 - 6 = \boxed{2}$$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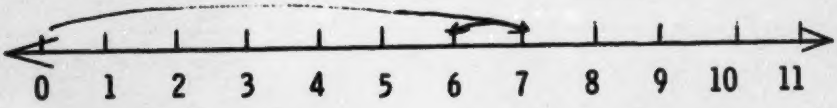
$$9 - 2 = \boxed{7}$$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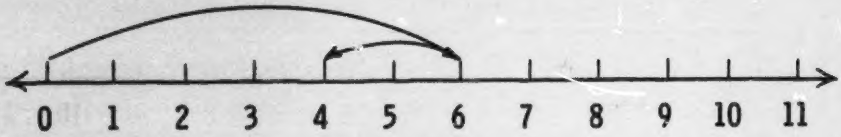
$$7 - 1 = \boxed{6}$$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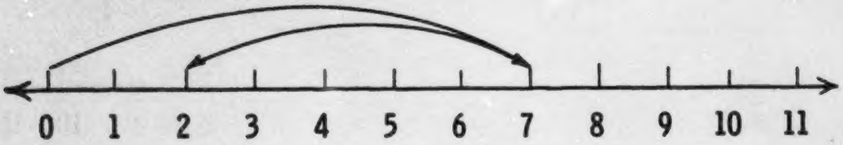
6 - 2 = 4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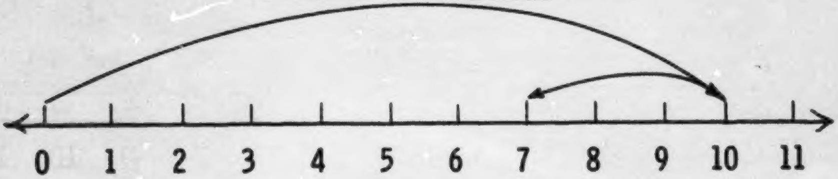
7 - 5 = 2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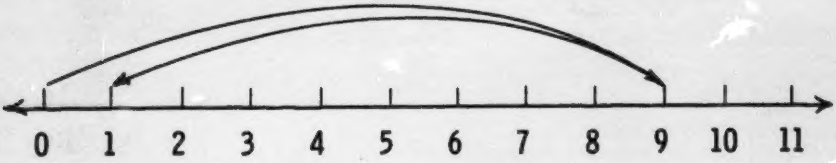
10 - 3 = 7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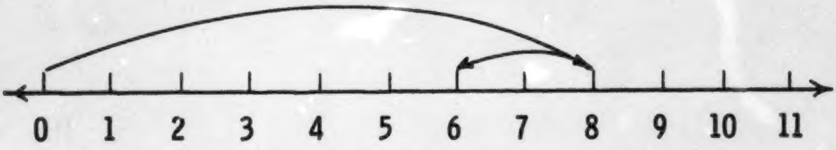
9 - 8 = 1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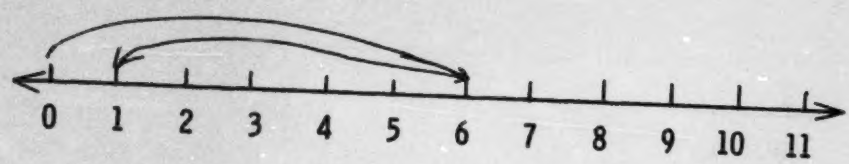
8 - 2 = 6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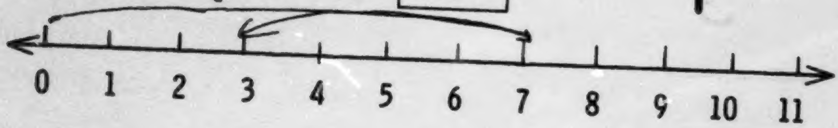
$6 - \boxed{5} = 1$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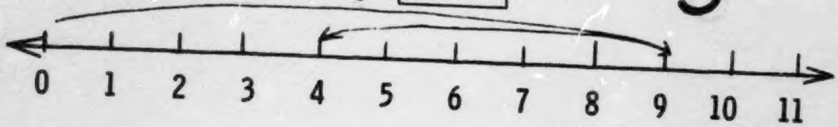
$7 - \boxed{4} = 3$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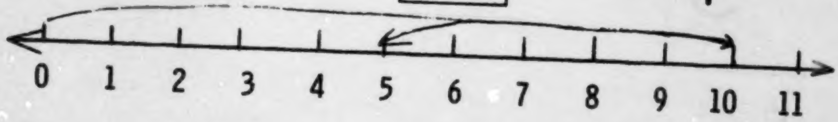
$9 - \boxed{5} = 4$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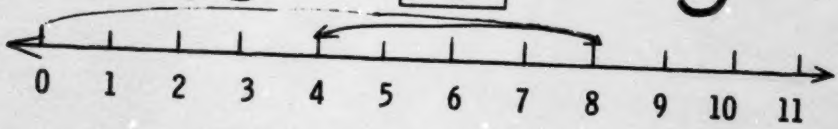
$10 - \boxed{5} = 5$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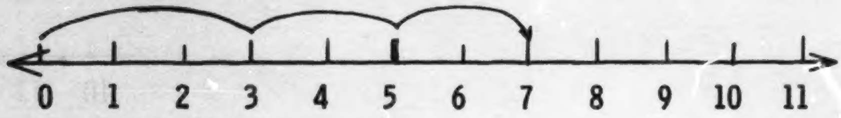
$8 - \boxed{4} = 4$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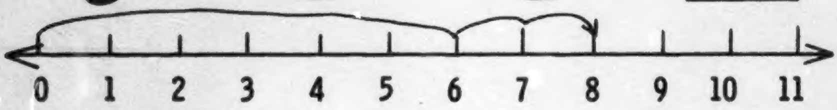
$3 + 2 + 2 = \boxed{7}$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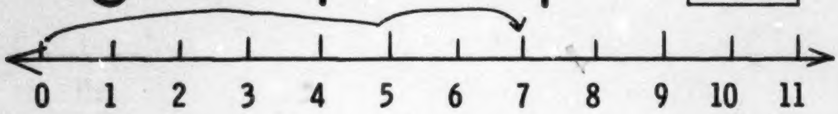
$6 + 1 + 1 = \boxed{8}$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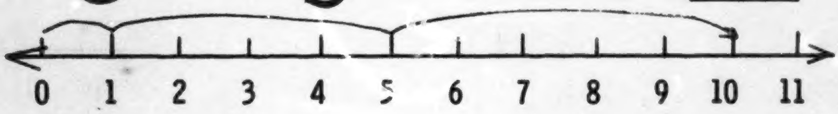
$5 + 0 + 2 = \boxed{7}$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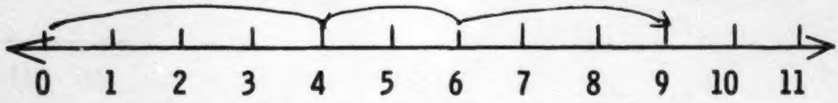
$1 + 4 + 5 = \boxed{10}$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$4 + 2 + 3 = \boxed{9}$

Chapter XI

PREVIEWS AND EXTENSIONS

XI-1. Partitions of sets 1 to 100

Objective: To reinforce the idea of partitioning through observing and recording partitions of sets of 1 through 100 into equivalent subsets of 2, 3, 4, 5, and 10

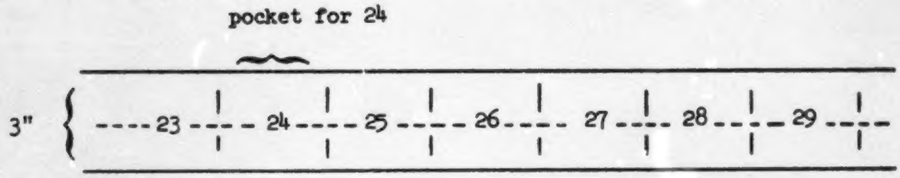
Vocabulary: No new terms

Materials: Flannel board and small counters to use on flannel board

Numeral cards 1 through 100

Hundred chart, made by children

Number line 1 through 100 with pockets behind the point indicated by the number. This can be done easily by mounting the number line on a bulletin board and stapling between and below the numbers; see illustration. Caution: do not use the region between the numbers to form the pocket.



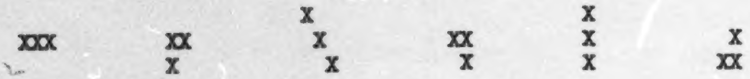
Markers for number line in green, orange, yellow, black, pink and purple. Strips should be approximately 4" × 1".

Suggested Procedure:

Select a numeral card, say 18. Ask a child to place that number of objects on the flannel board. This should be done with rows of tens and ones.

X X X X X X X X X X
X X X X X X X X

Briefly review with the children the meaning of equivalent sets. Ask a child to partition the set of 18 counters into equivalent sets with three in each subset. These may be placed in random positions on the flannel board.



How many equivalent sets do we have? (6)

Do we have a remainder set? (no)

We will call the set that remains after equivalent sets have been removed the remainder set. The number of the remainder set will be less than the number associated with the equivalent sets. Join the eighteen counters to make one set. Ask a child to partition the set into equivalent subsets with two in each subset. Join the eighteen counters to make one set. Ask a child to partition the set into equivalent subsets with two in each subset.

Do we have a remainder set? (No)

Continue to work with eighteen counters making equivalent subsets with four, five and ten members. When these have been finished recall with the children which types of partitions left no remainder set. Use a large chart similar to Pupil Page 91 and record the number of members in the set. Mark partitions which resulted in equivalent subsets with no remainder set by coloring the box red in the column below the number which describes the set. If there was a remainder set, color the box blue. See illustration.

Set	2	3	4	5	10
18	Red	Red	Blue	Blue	Blue
21	Blue	Red	Blue	Blue	Blue

Continue by using a set with 21 members. Ask a child to place the set on the flannel board by rows of tens and ones. (This is done to facilitate the teacher when children begin to work alone.) Children place their sets on their desks in a similar manner. The teacher can easily check to see that they have the correct number of members in their set before they begin to partition the set. Partition the set into equivalent subsets of 2, 3, 4, 5, and 10. This time the chart record can be made as the partition is completed.

Tell the children that if everyone works on a different number we will find out how all the sets 1 through 100 can be partitioned. Using Pupil Page 91 or a comparable sheet made by the teacher, give each child a numeral card and counters. He should first make a display with rows of tens and ones so the teacher can check to see that he has the correct number of members in the set to correspond with the numeral card. The teacher need not check the work of each child as he is making the partitions and recording his work, because a later culminating activity will provide an opportunity to check for errors.

When a child has finished the work with one set he should put his numeral card in a place where it will not be used by another child. He can then take a new card, get the necessary counters for that set, partition it into equivalent subsets and make a record of the work.

It may require several days to complete the work with the sets one to one hundred and each day's work should begin with a review of what is to be done with each set and how the work is to be recorded. The teacher may find that there will be some children who can work without the review lesson. If the teacher is confident that they know what to do, it would seem appropriate to let them begin alone.

The teacher may need to give additional help to children who have difficulty by "helping" them select their numeral cards. She may have to supervise the arranging of the first set of counters by some children. After all the sets have been partitioned, the entire set of individual charts is used in culminating the activity on the number line and hundred charts. A teacher may wish to use the first of these culminating activities or both.

Explain to the children that we will use the number line to show all of our work. Make a key for the number line which shows:

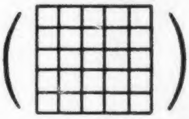
Equivalent subsets of 2	(with no remainder set)	green
Equivalent subsets of 3	(with no remainder set)	orange
Equivalent subsets of 4	(with no remainder set)	yellow
Equivalent subsets of 5	(with no remainder set)	black
Equivalent subsets of 10	(with no remainder set)	pink
Always a remainder set		purple

Colors should not be the same as those on individual charts.

Examine the group chart with the children. Look at the column of equivalent subsets with 2 members and find the boxes that are colored red. Place a green card at the point that numeral appears on the number line. Have the children look at their own charts and find the numbers which were partitioned into equivalent subsets with 2 members and had no remainder set by looking for the red boxes in the 2 column. Let the child then place a green marker at the point on the number line where that numeral appears. Next look at the column on the group chart which shows equivalent subsets with 3 members. Place an orange marker on the number line at these points. Ask children to look at the column on their own chart which shows the number of sets which were partitioned into equivalent subsets with no remainder set. The teacher may want to ask the children to point to the red boxes in the column and check to be certain they are using the chart in the right way. Let the children place an orange marker at these points on the number line. Show the children where this color of marker is indicated on the key. Continue by placing markers to show equivalent sets of 4, 5, and 10.

When the process has been completed, the purple markers can be placed in the spaces where there are no other markers. These should also correspond to rows on the charts which are all blue, because each of these sets always produced a remainder set when partitioned.

The culminating work on the number charts, 3' × 3' charts numbered to 100, is a follow up of the work on the number line.



Six number charts should be made, the frame drawn by the teacher and the numerals filled in by the children. This could be done at a time other than the math lesson and may meet with more careful workmanship if the children doing the work are told how this will be used. Committees with 2 members each could be used.

As the final activity, a committee of children colors on one chart all of the numerals green which have been marked with green strips on the number line. Another committee does the same with orange, yellow, black, pink and purple. When the hundred charts have been completed, the charts are studied to see patterns which occur within a given chart. Once the pattern is established they may see a number which does not fit the pattern, i.e., if 23 has been colored green, this indicates it can be divided into equivalent sets with 2 members in each subset and no remainder set. If it does not appear to fit the pattern of that chart it can be checked by working with that set on the flannel board. The teacher may find that discussing the patterns shown by 2, 5, and 10 is as far as her class is able to go. Work with the 3 and 4 pattern might be left to smaller groups.

Set	2	3	4	5	10

XI-2. Introducing odd and even numbers

Objective: To provide additional experience with counting by twos

Vocabulary: Odd, even

Materials: Number line
A cartooned cardboard insect

Suggested Procedure:

Children will have had experience counting by twos. Tell them the numbers 2, 4, 6, 8, 10, and so on, are called even numbers. The numbers 1, 3, 5, 7, 9, and so on, are called odd numbers. To give practice with these sets of numbers introduce an animated insect who steps two units at a time on the number line. He can go forward or backward. Place him on the number line and record his steps using even numbers and then using odd numbers. After some familiarity with the two sets of numbers is evident, pose questions such as these to the children: (for ease of questioning, give the insect a name)

If he begins on 21 will he take a step on 38 ? (No)

If he begins on 20 will he take a step on 44 ? (Yes)

If he begins on 18 could he step on 12 by going backward? (Yes)

What steps will he take going from 72 to 86 ? (74, 76, 78, 80, 82, 84)

What steps will he take going from 51 to 77 ? (53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75)

If he starts on 5 will he step on the set of even numbers or the set of odd numbers? (odd numbers)

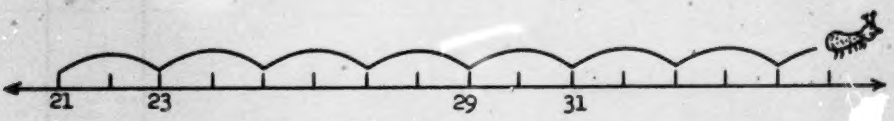
Which set of numbers, even or odd, has he used if he stops on 96 ? (even)

If he starts on 31, where will he be after he has taken 2 steps? (35)

If he starts on 63 where will he be if he takes 3 steps backward? (57)

Further activities:

Ditto copies of the number line showing the steps he made. Have children write the numeral not provided.



Ditto pairs of numerals and ask children to make a ring around the pair if the insect would land on both numerals during a walk along the number line.

[✓ 21 27]   [✓ 82 98]   14 21   [✓ 9 29]

[✓ 43 41]   70 53   35 36   [✓ 0 100]

Make a list of numerals. Ask the children to ring with blue the numerals for the numbers he would step on if he started on 0 and went walking only on even numbers. Ring with red the numerals for the numbers he would step on if he started at 1 and walked only on the odd numbers.

XI-3. Pre-multiplication and division

Objective: To work with equivalent sets and to partition sets into equivalent sets with remainder sets of 0 or more, to provide experiences related to multiplication and division concepts.

Vocabulary: No new terms

Materials: Flannel board and flannel board materials

Suggested Procedure:

Many situations will occur in the classroom which provide an opportunity for acquainting children with observations and vocabulary which build toward multiplication and division concepts. This lesson is presented to illustrate examples of possible classroom situations that might lead to these observations and the mathematical language which would describe the situations. Children should begin to note instances of equivalent sets and to describe the sets with numbers, i.e., there are 14 sets of blocks with 6 in each set.

Many classrooms would have boxes of eight crayons furnished to the children. These crayons may be described as sets of eight crayons. Thus the class may have 31 sets of crayons with eight members in each set. If some children have sets with a different number of members it might be interesting to note that we have 23 sets of children with eight members in their sets of crayons and 8 children with sixteen members in their sets of crayons, etc.

Other experiences can be given in a more formal way by using the flannel board and flannel board materials. Place a number of collections of sets on the flannel board and ask the children if the sets are equivalent. Use both equivalent and non-equivalent sets of like objects, and also both equivalent and non-equivalent sets of unlike objects. Fluorescent light fixtures in the classroom, library, hallway or auditorium of the school are possible examples of equivalent sets in that each fixture usually contains three to six tubes. Children could count how many sets of tubes were in the room.

Children play many games and work on projects in which there are sets of 2, 3, or 4 members. There are also times when there are 2, 3, or 4 sets of children with equivalent membership in each of the sets. In a mathematics lesson it would be interesting to recall with children situations when one of these organizational patterns occurs. A chart can be developed using these ideas:

Sets of 2	Sets of 3	Sets of 4
2 Sets	3 Sets	4 Sets

If children have access to a school library and check out a limited number of books at any one time, it could be noted after a checkout period that the sets of books we brought back to the room can be described as 27 sets of books with 2 members in each set. If some children check out 1, others 2 and still others 3, it would be possible to record the number of sets of 1 book, the number of sets of 2 books, and the number of sets of 3 books, etc. Many classrooms have window sashes which have equivalent sets of panes in each window. The children can tell how many sets of window panes are in the room.

If the children are planning to go on a field trip and the class must be grouped so that an adult accompanies each set of 5 or 6 children, the children can be given the number of adults that will make the visit with them and determine how many will go with each adult. This may result in a number of sets with 5 members and a number of sets with 6 members since all children will have to be included.

In preparing for class projects for social studies, science or art, in which 4 members are needed for each committee, the children can determine how many sets they will have; or if a given number of sets is desired, they can figure out how many members will participate in each group. In preparing for games in which teams or groups are needed, the children can determine how many will make up each team or how many teams will be used before starting the game.

If a more formal approach is desired, some experiences with flannel board materials can be used. Place a set with 24 members on the flannel board. Ask a child to remove a set with four members. Ask another child to remove subsets of the same number of members until the set has been partitioned into as many subsets with four members as can be made. Discuss how these sets are alike. What do we call sets which are alike in the number of members in the subsets when

304

it has been partitioned? (equivalent sets)

We can describe this partition by saying that 24 can be partitioned into 6 sets with 4 members in each subset. If the subsets have 5 members each, we would have said, "Twenty-four can be partitioned into four sets with five members in each set and a remainder set with four members."

XI-4. Another numeration system (enrichment material)

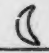
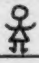
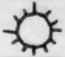
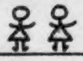
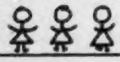
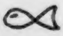
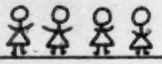
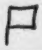
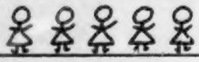
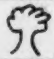
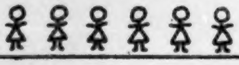
Objective: To deepen the understanding of place value for able children.

Teaching Notes:

This section really amounts to a rather elaborate game which is designed for rapidly moving students to deepen their understanding. It should not be attempted unless several days are available.

Suggested Procedure:

Explain to the children that different people sometimes use different names for numbers. In this lesson very special names are going to be used for some numbers. These names will be used to count and write the numbers. Begin with counting sets (avoid all mention of the usual number names), either using the special names given below, or getting the class to invent its names (Caution--if they select names, be sure that the names are short and that there are simple pictographs which can be used for numerals.)

	spoken name	numeral
empty set	moon	
	sun	
	ball	
	fish	
	flag	
	tree	
	zig	(base)

Tell the children that these are to be the only names for numbers that are to be used, and that they are going to see how well they can manage with just these. Give the children a little practice using the new names.

How many fingers do I have on my hand? (tree)
How many feet do I have? (ball)
What is sun plus ball? (fish)

After some preparation of this sort, show the children a set of 13 objects on the flannel board, and ask them how they could indicate, using our special numerals, the number of members in this set. Explore their ideas carefully. Probably some child will suggest "zig zig sun" as a possible name; if so, point out that "zig zig sun" is an absolutely correct way of naming the number. Remind the children that we need some way to write this number. Using a minimum of suggestion, try to get them to consider a pattern of the following sort:

zigs	suns

so that the set on the flannel board would correspond to:

zigs	suns
◍	☼

Proceed from listing in this fashion to the place value convention: ◍☼ is to mean ◍ zigs and ☼ suns. Continue with examples of this sort of numeration. Thus ⚑☼ is ⚑ zigs and ☼ suns, 🐟☾ is 🐟 zigs and ☾ suns, and so on. Be sure, at this stage, to consider only sets with fewer than 36 members.

Use the "ig" suffix on the word when it is in the zig position, thus, sunig, ballig, fishig, flagig, treeig. For sets which reach zigs zigs the term used could be willy, then the grouping is by:

willys	zigs	suns

◍🐟☼ is then read "ballwilly fishig sun". It is suggested that actual work with this type of material would end when the ☼☾☾ was reached. (Good luck!)

XI-5. The union of three sets; the sum of three numbers

Objective: To teach the process of joining three sets and the process of adding three numbers. To introduce the use of parentheses.

Vocabulary: Parentheses, (optional: associative)

Materials: Flannel board objects, set materials

Suggested Procedure:

Arrange on the flannel board a set of three objects at the left, a set of four in the middle, and a set of two objects to the right. Elicit from the class the facts that there are 3 objects in the left hand set, 4 in the middle set, and 2 in the right hand set. Record these facts on the chalkboard:

3 4 2

Explain that the children are going to find out how many members are in the union of all three of the sets by addition, without counting. Say that they can start with the left hand sets; and with yarn, indicate the union of these two sets. Bring out the fact that the set inside the yarn is the union of a set of 3 and a set of 4 and therefore has 3 + 4, which is 7, members. Record this information on the chalkboard, so that the chalkboard shows:

3 4 2
7 2

Now bring out the fact that the union of the three sets is the same as the union of the set inside the yarn and the right hand set, and therefore has 7 + 2 or 9 members. Record this, so that the chalkboard shows:

3 4 2
7 2
9

Say that there are 9 members in the union of all three sets, and that this was obtained by adding 3 and 4, and adding 2 to the result.

Next ask the class what equation describes what was done, and try to get as far as: 3 + 4 + 2 = 9. Accept this as correct. Then ask how this equation could be changed to show that first 3 and 4 were added, and lead up to the notion of parentheses, possibly through the following steps:

$$\boxed{3 + 4} + 2 = 9, \quad \boxed{3 + 4} + 2 = 9, \quad (3 + 4) + 2 = 9.$$

Next repeat this procedure, beginning with the two right hand sets, and arriving at: $3 + (4 + 2) = 9$. Finally, point out that the number of members in the union of the three sets does not depend on the way that the calculation was made, so that of course

$$(3 + 4) + 2 \text{ is equal to } 3 + (4 + 2).$$

(If you wish, tell them that this is a case of the associative law, or that addition is called associative because it behaves this way.)

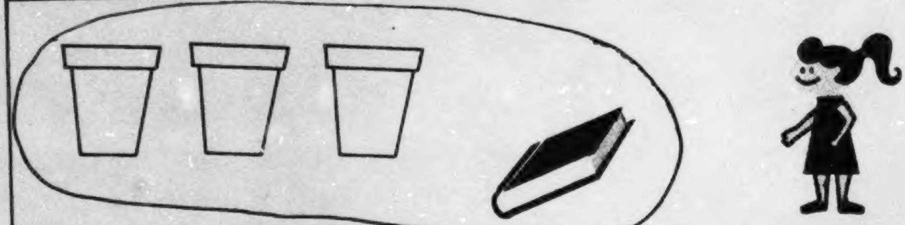
Next, start with an equation, say $(1 + 4) + 3 = 8$, and get them to try to imagine sets that go with this equation (a set of 1 on the left and a set of 4 in the middle, with yarn circling these, and a set of 3 to the right). Then change the problem by writing the equation: $1 + (4 + 3) = 8$. Further exploration might lead to other sorts of grouping.

Pupil Page 92: Draw a ring around the first and second set in each box. Write the equation in the proper spaces below.

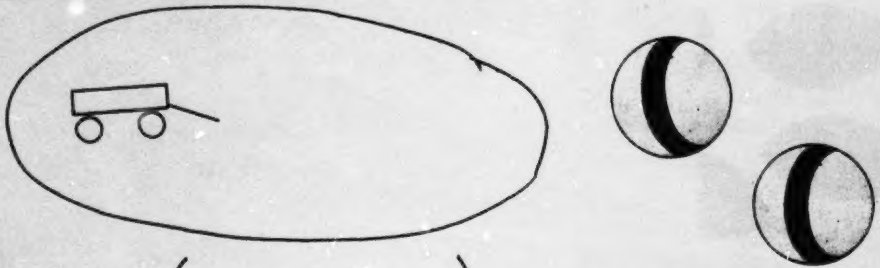
Pupil Page 93: Draw a ring around the second and third sets in each box. Write the equation in the proper spaces below.

Pupil Pages 94-95: Draw a picture for each set joined together as suggested by the equation in each box. Place a ring around the two sets as indicated by the parentheses in the equation.

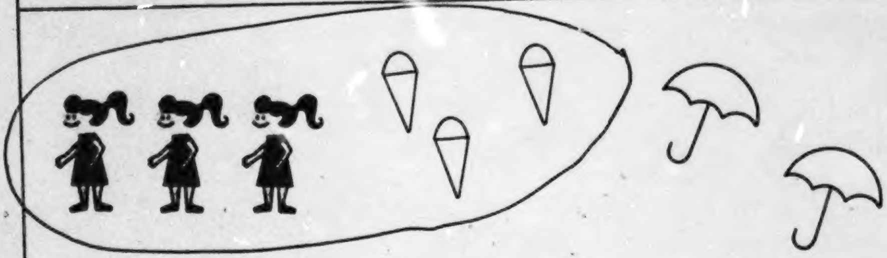
Pupil Pages 96-99: Fill in the blanks to complete the equations.



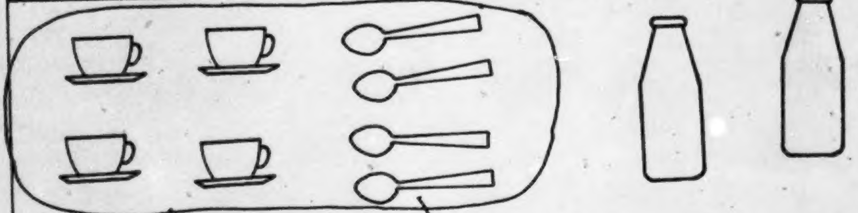
$(3+1)+1=5$



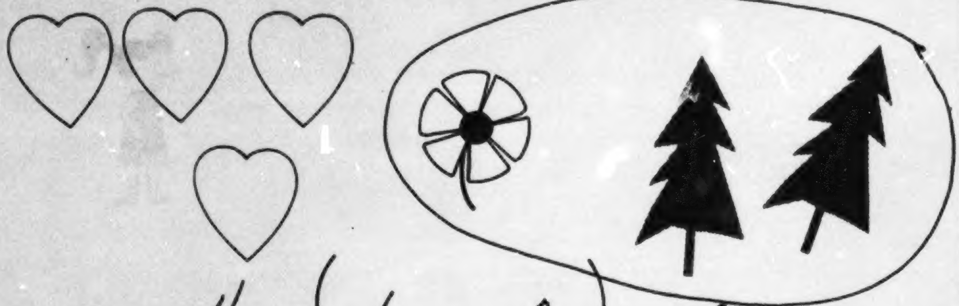
$(\underline{1} + \underline{0}) + \underline{2} = \underline{3}$



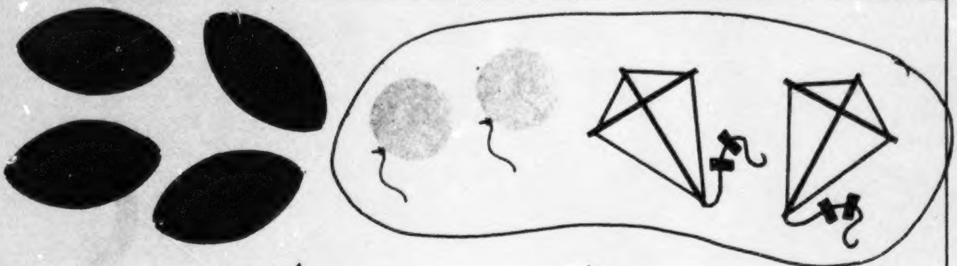
$(\underline{3} + \underline{3}) + \underline{2} = \underline{8}$



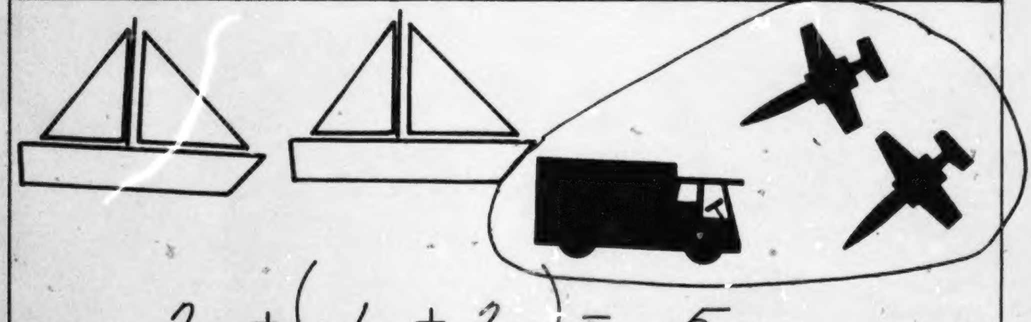
$(\underline{4} + \underline{4}) + \underline{2} = \underline{10}$



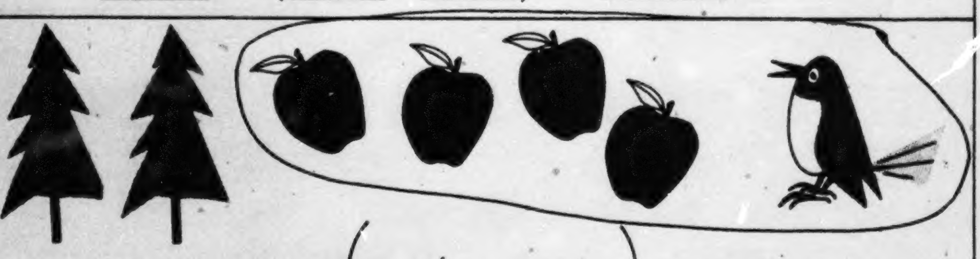
4 + (1 + 2) = 7



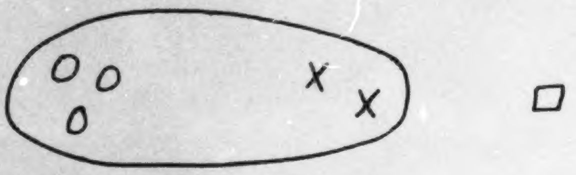
4 + (2 + 2) = 8



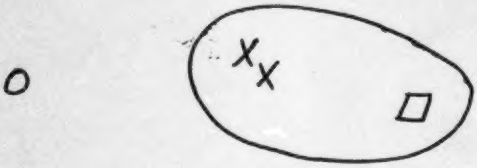
2 + (1 + 2) = 5



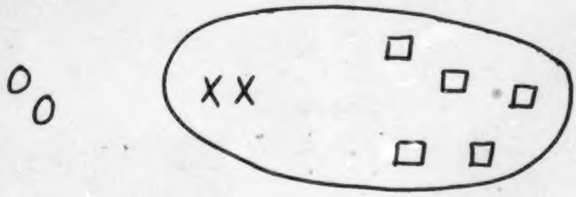
2 + (4 + 1) = 7



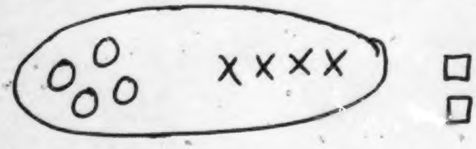
$(3+2)+1=$ 6



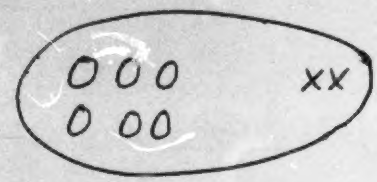
$1+(2+1)=$ 4



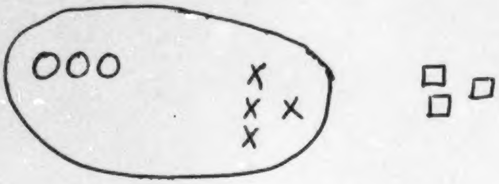
$2+(2+5)=$ 9



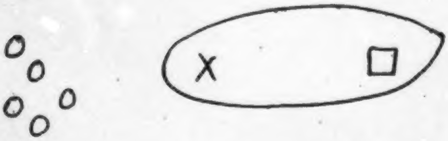
$(4+4)+2=$ 10



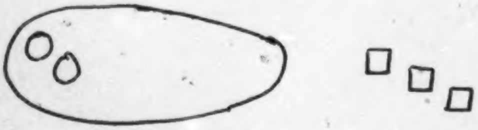
$(6 + 2) + 0 =$ 8



$(3 + 4) + 3 =$ 10



$5 + (1 + 1) =$ 7



$(2 + 0) + 3 =$ 5

$2+(3+4)=$ 9

$(1+5)+2=$ 8

3 $+(3+2)=8$

$(1+2)+6=$ 9

$(2+2)+$ 2 $=6$

$(3+0)+2=$ 5

$6+(2+1)=$ 9

$0+(1+2)=$ 3

$(3+2)+1=$ 6

$3+(2+1)=$ 6

$2+(1+3)=$ 6

$(4+2)+3=$ 9

$4+(2+3)=$ 9

$(3+$ 4 $)+2=9$

$(5+1)+4=$ 10

5 $+(1+4)=10$

3 + 4 + 1 = 8

6 + 1 + 2 = 9

5 + 2 + 2 = 9

4 + 4 + 0 = 8

1 + 4 + 4 = 9

3 + 3 + 3 = 9

2 + 3 + 2 = 7

1 + 4 + 3 = 8

1 + 0 + 8 = 9

4 + 3 + 2 = 9

1 + 1 + 7 = 9

5 + 0 + 4 = 9

2 + 1 + 6 = 9

2 + 4 + 3 = 9

3 + 5 + 3 = 11

5 + 2 + 2 = 9